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EUROPEAN COMMISSION
DIRECTORATE-GENERAL
REGIONAL AND URBAN POLICY

Guidance on sampling methods for audit authorities

(Under Article 62 of Council Regulation (EC) N° 1083/2006, Article 16, including Annex IV, of Commission Regulation (EC) N° 1828/2006 and Articles 61 of Council Regulation (EC) N° 1198/2006, Article 42, including Annex IV, of Commission Regulation (EC) N° 498/2007)

DISCLAIMER:

"This is a working document prepared by the Commission services. On the basis of the applicable EU law, it provides technical guidance to the attention of public authorities, practitioners, beneficiaries or potential beneficiaries, and other bodies involved in the monitoring, control or implementation of the Cohesion policy on how to interpret and apply the EU rules in this area. The aim of this document is to provide Commission services' explanations and interpretations of the said rules in order to facilitate the implementation of operational programmes and to encourage good practice(s). However this guidance is without prejudice to the interpretation of the Court of Justice and the General Court or decisions of the Commission."

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List of Acronyms

AA – Audit Authority

ACR – Annual Control Report

AE – Anticipated Error

AR – Audit Risk

BP – Basic Precision

BV – Book Value (expenditure certified to the Commission in reference year)

COCOF – Committee of the Coordination of Funds

CR – Control Risk

DR – Detection Risk

E_i – Individual errors in the sample

$\bar{E}$ – Mean error of the sample

EC – European Community

EE – Projected Error

EDR – Extrapolated Deviation Rate

EF – Expansion Factor

ETC – European Territorial Cooperation

IA – Incremental Allowance

IR – Inherent Risk

IT – Information Technologies

MCS – Managing and Control System

MUS – Monetary Unit Sampling

PPS – Probability Proportional to Size

RF – Reliability Factor

SE – (Effective, i.e., after performing audit work) Sampling Error (precision)

SI – Sampling Interval

TE – Maximum Tolerable Error

TPER - Total Projected Error Rate

TPE – Total Projected Error

ULD – Upper Limit of Deviation

ULE – Upper Limit of Error

1 Introduction

The present guide to statistical sampling for auditing purposes has been prepared with the objective of providing audit authorities in the Member States with an updated overview of the most commonly used and suitable sampling methods, thus providing support for the implementation of the regulatory framework for the current and, where applicable, the next programming period.

The selection of the most appropriate sampling method to meet the requirements of Article 62 of Council Regulation (EC) N° 1083/2006, Article 16, including Annex IV, of Commission Regulation (EC) N° 1828/2006 and Articles 61 of Council Regulation (EC) N° 1198/2006, Article 42, including Annex IV, of Commission Regulation (EC) N° 498/2007is at the audit authority's own professional judgement. Accordingly, this guidance seeks to aid audit authorities in the implementation of the audit work.

The selected method should be described in the audit strategy referred to in Article 62 (1) (c) of Regulation N° 1083/2006 and Article 61 of Regulation (EC) N° 1198/2006 which should be established in line with model of Annex V of the Commission Regulations (EC) N° 1828/2006 and N° 498/2007 and any change in the method should be indicated in subsequent versions of the audit strategy and transmitted to the Commission in the next Annual Control Report.

International auditing standards and updated sampling theory provide guidance on the use of audit sampling and other means of selecting items for testing when designing audit procedures.

The present guidance replace the previous guidance on the same subject (ref. COCOF 08/0021/02-EN of 15/09/2008). However, the present document is without prejudice of other complementary Commission guidelines, namely the:

- “Guidance note on annual control reports and opinions” of 18/02/2009, ref COCOF 09/0004/01-EN and EFFC/0037/2009-EN of 23/02/2009;
- “Guidance on treatment of errors disclosed in the annual control reports” of 07/12/2011, ref COCOF 11/0041/01-EN and EFFC/87/2012 of 09/11/2012;
- “Guidance on a common methodology for the assessment of management and control systems [MSC] in the Member States” ref COCOF 08/0019/01EN and EFFC/27/2008 of 12/09/2008.

Thus, complementary reading of these additional documents is advised in order to get a complete view of the guidelines related to the production of annual control reports.

2 Reference to the legal basis – regulatory framework

Article 62(1)(a)&(b) of Council Regulation (EC) No 1083/2006 of 11 July 2006 laying down the general provisions of the European Regional Development Fund, the European Social Fund and the Cohesion Fund and Article 61 of Regulation (EC) N° 1198/2006 of 27 July 2006 laying down the general provisions of the European Fisheries Fund refers to the responsibility of the audit authority to ensure the execution of audits of the management and control systems and of audits of operations on the basis of an appropriate sample.

Commission Regulation (EC) No 1828/2006 of 8 December 2006 and N° 498/2007 of 26 March 2007 (hereafter "the Regulations") setting out rules for the implementation of Council Regulation (EC) No 1083/2006 and N° 1198/2006 establish detailed provisions in relation to sampling for audits of operations in Articles 16[1], 17[2] and Annex IV, and in Articles 42, 43 and Annex IV, respectively.

The Regulations define the requirements for the system audits and audits of operations to be carried out in the framework of the Structural Funds, and the conditions for the sampling of operations to be audited which the audit authority has to observe in establishing or approving the sampling method. They include certain technical parameters to be used for a random statistical sample and factors to be taken into account for a complementary sample.

The principal objective of the systems audits and audits of operations is to verify the effective functioning of the management and control systems of the operational programme and to verify the expenditure declared[3].

The Regulations also set out the timetable for the audit work and the reporting by the audit authority.

The audits of operations are carried out on the expenditure declared to the Commission in the reference year (random sample reference period). In order to provide an annual opinion, the audit authority should plan the audit work, including systems audits and audits of operations, properly.

[1] Article 16.1 states "The audits referred to in point (b) of Article 62(1) of Regulation (EC) No 1083/2006 shall be carried out each twelve-month period from 1 July 2008 on a sample of operations selected by a method established or approved by the audit authority in accordance with Article 17 of this Regulation":

[2]Article 17.2 states "The method used to select the sample and to draw conclusions from the results shall take account of expenditure, the number and type of operations and other relevant factors, the audit authority shall determine the appropriate statistical sampling method to apply. The technical parameters of the sample shall be determined in accordance with Annex IV."

[3] Article 62 (1) (c) of Council Regulation (EC) No 1083/2006 (OJ L210/25) and Article 61 (1) (c) of Regulation (EC) N° 1198/2006 (OJ L223/1)

3 Audit risk model and audit procedures

3.1 Risk model

Audit risk is the risk that the auditor issues an unqualified opinion, when the declaration of expenditure contains material errors.

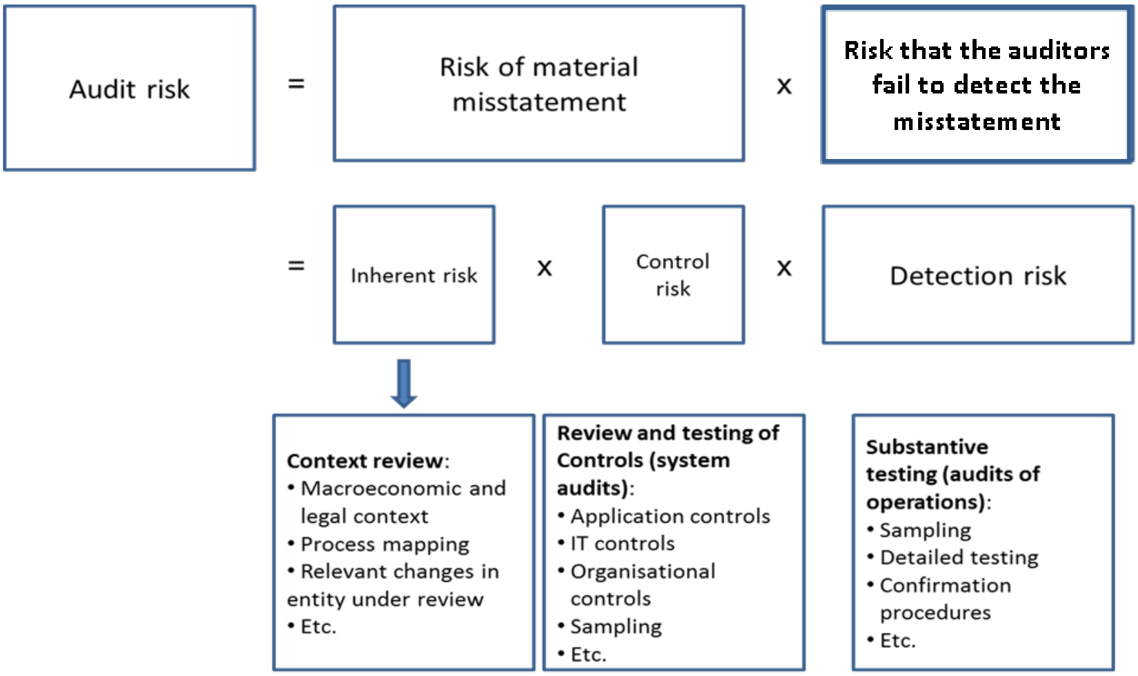


Fig 1. Audit risk model

The three components of audit risk are referred to respectively as inherent risk (IR), control risk (CR) and detection risk (DR). This gives rise to the audit risk model

$$AR = IR \times CR \times DR$$

where:

- IR, inherent risk, is the perceived level of risk that a material error may occur in the certified statements of expenditure to the Commission, or underlying levels of aggregation, in the absence of internal control procedures. The inherent risk is linked to the kind of activities of the audited entity and will depend on external factors (cultural, political, economic, business activities, clients and suppliers, etc.) and internal factors (type of organisation, procedures, competence of staff, recent changes to processes or management positions, etc.). IR risk needs to be assessed before starting detailed audit procedures (interviews with management and key personnel, reviewing contextual information such as organisation charts, manuals and internal/external documents). For the Structural and Fisheries Funds, the inherent risk is usually set at a high percentage.
- CR, control risk, is the perceived level of risk that a material error in certified statements of expenditure to the Commission, or underlying levels of

aggregation, will not be prevented, detected and corrected by the management's internal control procedures. As such the control risks are related to how well inherent risks are managed (controlled) and will depend on the internal control system including application controls, IT controls and organisational controls, to name a few. Control risks can be evaluated by means of **system audits** - detailed tests of controls and reporting, which are intended to provide evidence about the effectiveness of the design and operation of a control system in preventing or detecting material errors and about the organisation's ability to record, process, summarize and report data.

The product of inherent and control risk (i.e. $IR \times CR$) is referred to as the **risk of material error**. The risk of material error is related to the result of the **system audits**.

- DR, detection risk, is the perceived level of risk that a material error in the certified statements of expenditure to the Commission, or underlying levels of aggregation, will not be detected by the auditor. Detection risks are related to how adequately the audits are performed, including sampling methodology, competence of staff, audit techniques, audit tools, etc. Detection risks are related to performing audits of operations. This includes substantive tests of details or transactions relating to operations in a programme, usually based on sampling of operations.

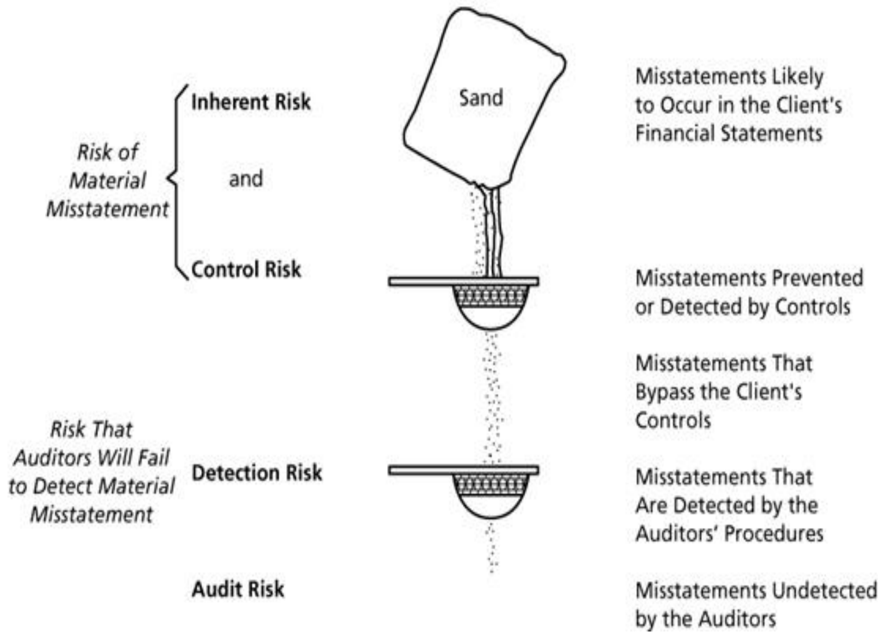


Fig. 2 Illustration of audit risk (adapted from an unknown source)

The assurance model is the opposite of the risk model. If the audit risk is considered to be 5%, the audit assurance is considered to be 95%.

The use of the audit risk/audit assurance model relates to the planning and the underlying resource allocation for a particular operational programme or several operational programmes and has two purposes:

- Providing a high level of assurance: assurance is provided at a certain level, e.g. for 95% assurance, audit risk is then 5%.
- Performing efficient audits: with a given assurance level of for example 95%, the auditor should develop audit procedures taking into consideration the *IR* and *CR*. This allows the audit team to reduce audit effort in some areas and to focus on the more risky areas to be audited.

Note that the setting of the detection, which in turn controls the sample size for the sampling of operations, is a straightforward result, provided that the *IR* and the *CR* have been previously assessed. In fact,

$$AR = IR \times CR \times DR \Longrightarrow DR = \frac{AR}{IR \times CR}$$

where the AR is usually set to 5%, IR and CR are assessed by the auditor.

Illustration

Low control assurance: Given a desired, and accepted audit risk of 5%, and if inherent risk (=100%) and control risk (= 50%) are high, meaning it is a high risk entity where internal control procedures are not adequate to manage risks, the auditor should strive for a very low detection risk at 10%. In order to obtain a low detection risk the amount of substantive testing and therefore sample size need to be large.

$$AR = IR \times CR \times DR = 1 \times 0.5 \times 0.1 = 0.05$$

High control assurance: In a different context, where inherent risk is high (100%) but where adequate controls are in place, one can assess the control risk as 12.5%. To achieve a 5% audit risk level, the detection risk level can be at 40%, the latter meaning that the auditor can take more risks by reducing the sample size. In the end, this will mean a less detailed and a less costly audit.

$$AR = IR \times CR \times DR = 1 \times 0.125 \times 0.4 = 0.05$$

Note that both examples result in the same achieved audit risk of 5% within different environments.

To plan the audit work, a sequence should be applied in which the different risk levels are assessed. First, the inherent risk needs to be assessed and, in relation to this, control risk needs to be reviewed. Based on these two factors, the detection risk can be set by the audit team and will involve the choice of audit procedures to be used during the detailed tests.

However, the audit risk model provides a framework for reflection on how to construct an audit plan and allocate resources, in practice it may be difficult to quantify precisely inherent risk and control risk.

Assurance/confidence levels for the audit of operations depend mainly on the quality of the system of internal controls. Auditors evaluate risk components based on knowledge and experience using terms such as LOW, MODERATE/AVERAGE or HIGH rather than using precise probabilities. If major weaknesses are identified during the systems audit, the control risk is high and the assurance level obtained from the system would be low. If no major weaknesses exist, the control risk is low and if the inherent risk is also low, the assurance level obtained from the system would be high.

In the context of the Structural and Fisheries Funds, Annex IV of the Regulations (state that: "In order to obtain a high level of assurance, that is, a reduced audit risk, the audit authority should combine the results of system audits (*which corresponds to the control assurance*) and audits of operations (*detection assurance*). The combined level of assurance obtained from the systems audits and the audits of operations should be high. The audit authority should describe in the annual control report the way assurance has been obtained". It is expected that the audit authority needs to obtain a 95% level of assurance in order to be able to state that it has "reasonable assurance" in its audit opinion. Accordingly, the audit risk is 5%. The assumption contained in the Regulations is that even a poorly functioning system will always give a minimum assurance (i.e. a risk of material misstatement not larger than 50%) and that the remaining assurance (90%) is obtained from the audit of operations.

In the exceptional case that the audit authority concludes that no assurance at all can be obtained from the system, the assurance/confidence level to be obtained from the audit of operations is 95%.

As previously indicated, if major weaknesses are identified during the systems audit, one can say that the risk of material error is high (control risks in combination with inherent risks) and as such the assurance level given by the system would be low. Annex IV of the Regulations indicates that if the assurance level obtained from the system is low the confidence level to be applied for sampling of operation would be not less than 90%.

If no major weaknesses in the systems exist the risk of material errors is low, and the assurance level given by the system would be high meaning that the confidence level to be applied for sampling of operations would be not less than 60%.

Section 3.2 provides a detailed framework for choosing the assurance/confidence level for the audit of operations.

3.2 Assurance/confidence level for the audit of operations

Annex IV of the Regulations states that substantive tests should be performed on samples, the size of which will depend on a confidence level determined according to the assurance level obtained from the system audit, i.e.

- not less than 60% if assurance is high;
- average assurance (no percentage corresponding to this assurance level is specified in the Commission Regulation although a 70% to 80% of assurance is advised);
- not less than 90% if assurance is low.

Annex IV also states that the audit authority shall establish criteria used for system audits in order to determine the reliability of the management and control systems. These criteria should include a quantified assessment of all key elements of the systems (key requirements) and encompass the main authorities and intermediate bodies participating in the management and control of the operational programme.

The Commission in collaboration with the European Court Auditors has developed a guidance note on the methodology for the evaluation of the management and control systems. It is applicable both to mainstream and ETC programmes. It is recommended that the audit authority take account of this methodology.

In this methodology, four reliability levels are foreseen:
- Works well, only minor improvements are needed
- Works, but some improvements are needed
- Works partially, substantial improvements are needed
- Essentially does not work.

In accordance with the Regulation, the confidence level for sampling is determined according to the reliability level obtained from the system audits.

As indicated above, the Regulation foresees only three levels of assurance on systems: high, average and low. The average level effectively corresponds to the second and third categories of the methodology for evaluation of the management and control systems, which provide a more refined differentiation between the two extremes of high/“works well” and low/“does not work”.

The recommended relationship is shown in the table below:

Assurance level from the system audits	**Related reliability in the Regulation/assurance from the system**	**Confidence level**	**Detection Risk**
Works well, only minor improvements are needed	High	Not less than 60%	Less or equal to 40%
Work, but some improvements are needed	Average	70%	30%
Works partially, substantial improvements are needed	Average	80%	20%
Essentially does not work	Low	Not below 90%	Not greater than 10%

Table 1. Confidence level for the audit of operations according to the assurance from the system

It is expected that at the beginning of the programming period, the assurance level is low as no or only a limited number of system audits will have taken place. The confidence level to be used would therefore be not less than 90%. However, if the systems remain unchanged from the previous programming period and there is reliable audit evidence on the assurance they provide, the Member State could use another confidence level (between 60 % and 90 %). The confidence level can also be reduced during a programming period if no material errors are found or there is evidence that the systems have been improved over time. The methodology applied for determining this confidence level will have to be explained in the audit strategy and the audit evidence used to determine the confidence level will have to be mentioned.

Setting an appropriate confidence level is a critical issue for the auditing of operations, as sample size is strongly dependent on this level (the higher the confidence level the larger the sample size). Therefore the regulations offer the possibility of reducing the confidence level and consequently audit workload for systems with a low error rate (therefore high assurance), while maintaining the requirement of a high confidence level (consequently larger sample size) in the case of a systems that has a potentially high error rate (therefore low assurance).

Determination of the applicable assurance level when grouping programmes

The audit authority should apply **one** assurance level in the case of grouping of programmes.

In case the system audits reveal that within the group of programmes there are differences in the conclusions on the functioning of the various programmes, the following options are available:

- to create two (or more) groups, for example the first for programmes with a low level of assurance (confidence level of 90%), the second group for programmes with a high level of assurance (a confidence level of 60%), etc. The two groups are treated as two different populations. Consequently the number of controls to be performed will be higher, as a sample from each separate group will have to be taken;
- to apply the lowest assurance level obtained at the individual programme level for the whole group of programmes. The group of programmes is treated as one single population. In this case, audit conclusions will be drawn to the whole group of programmes. Consequently, conclusions about each individual program will not usually be possible.

In the latter case, it is possible to use a sampling design stratified by programme, which will usually allow a smaller sample size. Nevertheless, even when using stratification a single assurance level have to be used and conclusions are still only possible for the whole group of programmes.

4 Treatment of errors

As stated before, this document should be read together with the “Guidance on treatment of errors disclosed in the annual control report” where a detailed presentation of several types of errors is offered. Thus, this section does not intend to exhaustively reproduce the detailed analysis and definitions for the several types of errors, but only to present a brief summary of topics that have direct impact on sampling methodology and the projection of total error.

4.1 Systemic and known errors

Systemic errors

The systemic errors are errors found in the sample audited that have an impact in the non-audited population and occur in well-defined and similar circumstances. These errors generally have a common feature (e.g. type of operation, intermediate body, location, or period of time). They are in general associated with ineffective control procedures within (part of) the management and control systems. Indeed, the identification of a potential systemic error implies carrying out the complementary work necessary for the identification of its total extent and subsequent quantification. This means that all the situations susceptible of containing an error of the same type as the one detected in the sample should be identified, thus allowing the delimitation of its total effect in the population.

If the AA has reasonable assurance that the subpopulation affected by systemic errors is fully delimited and there are no other units in the population susceptible to be affected by similar errors, the amount of systemic errors should be added[4] to the random projected error and to the anomalous uncorrected errors in order to produce the total error. When extrapolating the random errors found in the sample to the population, the AA should take into account that the random errors are extrapolated only to the remaining expenditure (total expenditure deducted from the amount of systemic errors). This is achieved by transforming, whenever necessary, the formulas used for projecting the errors and calculating precision as showed in appendix 1. The amount of systemic errors found in the sample is not considered as random error and is not accounted for in the random projected error. Nevertheless, any random errors found in the operations affected by systemic errors should be extrapolated and accounted for in the random projected error.

Known errors

It can also be that an error found in the sample leads the auditor to detect one or more errors outside that sample. These errors identified outside the sample are classified as "known errors". For example, if a contract is found to be illegal under the public procurement rules it is likely that part of the related irregular expenditure has been declared in a payment claim or invoice included in the sample audited and the remaining expenditure has been declared in payment claims or invoices not included in that sample. The main difference in relation to systemic errors is that the expenditure affected by the known error is typically circumscribed to one operation.

When the AA adds the known errors to the Total Projected Error Rate[5] (TPER - see below section 4.4), the AA should take into account that the random errors in the sample (including the error that led to the detection of the known error)[6] are extrapolated only to the remaining expenditure (total expenditure deducted from the amount of known errors). This is achieved by transforming, whenever necessary, the

[4] And not the amount of expenditure of the subpopulation to which the systemic errors relate to.

[5] As foreseen in the "Guidance on treatment of errors disclosed in the annual control reports (ref COCOF 11/0041/01-EN of 07/12/2011and EFFC/87/2012 of 09/11/2012).

[6] Contrary to systemic errors, the delimitation of known errors cannot ensure that there are no other operations affected by this type of error or other irregularity.

formulas used for projecting the errors and calculating precision as showed in appendix 1.

A simpler approach is to extrapolate the random errors in the sample (including the error that led to the detection of the known error) to the total expenditure (without deducting the amount of known errors). In this case, the known error is not added to the TPER.

4.2 Random errors

The errors that are not considered as systemic or known are classified as random errors. This concept presumes the probability that random errors found in the audited sample are also present in the non-audited population, since the sample is representative of the population. Hence, these errors are to be included in the calculation of the projection of errors.

4.3 Anomalous errors

An error that is demonstrably not representative of the population is called anomalous error. A statistical sample is representative for the population and therefore anomalous errors should only be accepted in very exceptional, well-motivated circumstances.

The frequent recourse to this concept without a due justification may undermine the reliability of the audit opinion.

The AA is required to provide in the ACR a high degree of certainty that such an anomalous error is not representative of the population (does not appear elsewhere in the population) and to explain the additional audit procedures it carried out to conclude on the existence of an anomalous error, as required by the ISA n° 530. The ISA n° 530 further specifies:

"A.19. When a misstatement has been established as an anomaly, it may be excluded when projecting misstatements to the population. However, the effect of any such misstatement, if uncorrected, still needs to be considered in addition to the projection of the non-anomalous misstatements".

A.22. In the case of tests of details, the projected misstatement plus anomalous misstatement, if any, is the auditor's best estimate of misstatement in the population. When the projected misstatement plus anomalous misstatement, if any, exceeds tolerable misstatement, the sample does not provide a reasonable basis for conclusions about the population that has been tested. (…)"

This means that, when the AA decides to exclude an anomalous error from the calculation of the projected error, the amount of the anomalous error is to be added in the calculation of the total projected error rate if it has not been corrected. One way to perform this calculation is to exclude the related operation both from the population and from the sample before calculating the projected error from the sample. After obtaining the projected error, the amount of the individual uncorrected anomalous error should be added in order to obtain the total projected error. If the anomalous error has been corrected then it does not count for the total projected error rate

4.4 Total projected error rate (TPER)

The AA should disclose in the ACR the total projected error rate, which the AA should compare with the materiality threshold in order to reach conclusions for the population, as follows from the second subparagraph of Article 17(4) of the Commission Regulation (EC) No 1828/2006 and 43 (4) of Commission Regulation (EC) No 498/2007. According to the 2nd paragraph of this provision, *"in operational programmes for which the projected error rate is above the materiality level, the audit authority shall analyse its significance and take the necessary actions, including making appropriate recommendations, which will be communicated in the annual control report"*.

The total projected error rate represents the estimated effect of the errors in the management and control systems, in percentage of the population. The total projected error should reflect the analysis done by the AA concerning the errors detected in the context of the audits of operations carried out under Article 62.1(b) of Regulation (EC) N° 1083/2006 and Article 61.1(b) of Regulation 1198/2006.

The total projected error (TE) corresponds to the sum of the following errors: projected random errors, systemic errors, and uncorrected anomalous errors. Known errors can also be added to the TE but a simpler approach is set out above in section 4.1.

If systemic errors are identified in the audited sample and their extension in the population not audited is delimited with precision, then the systemic errors relating to the population are added to the total projected error. If such delimitation is not done before the ACR is submitted, the systemic errors are to be treated as random for the purposes of the calculation of the projected random error.

Concerning random errors, the calculation of the projection of errors differs according to the sampling method selected. All errors should be quantified by the AA and included in the total projected error rate, with the exception of corrected anomalous errors. Without this quantification, the error rate cannot be considered reliable since it is probably understated. In general, all errors found are to be taken into account for calculation of the total projected error rate.

If the systemic error affects **expenditure declared in previous years** and those errors were not included in previously reported total projected error rates, the AA should revise those error rates accordingly. The implementation of the corrective measures may have an impact in this revision. If systemic or known errors are detected in the expenditure declared in the year N and also the expenditure declared the subsequent year(s), those errors should be taken into account in the calculation of the total projected error rate~~s~~ in the subsequent year(s).

As results from the guidance note on Annual Control Reports (COCOF 09/0004/01-EN of 18/02/2009 and EFF/0037/2009-EN), errors found in systems audits (control testing) are <u>not</u> added to the total projected error, but should be corrected and disclosed in section 4 of the ACR. Obviously, the conclusions drawn from systems audits should be taken into account in the audit opinion disclosed in the ACR, together with the outcome of the audits of operations.

5 Statistical concepts related to audits of operations

5.1 Sampling method

The sampling method encompasses two elements: the sampling design (e.g. equal probability, probability proportional to size) and the projection (estimation) procedure. Together, these two elements provide the framework to calculate sample size.

The most well know sampling methods suitable for the audit of operations are presented in Section 6.1. Please note that the first distinction between sampling methods is made between statistical and non-statistical sampling.

A statistical sampling method has the following characteristics:

- each item in the population has a known and positive selection probability;
- randomness should be ensured by using proper random number generating software, specialised or not (e.g. MS Excel provides random numbers).

Statistical sampling methods allow the selection of a sample that is "representing" the population (reason why statistical selection is so important). The final goal is to project (extrapolate or estimate) to the population, the value of a parameter (the "variable") observed in a sample, allowing to conclude whether a population is materially misstated or not and, if so, by how much (an error amount).

Non-statistical sampling does not allow the calculation of precision, consequently there is no control of the audit risk and it is impossible to ensure that the sample is representing the population. Therefore, the error has to be assessed empirically.

Statistical sampling is required by Council Regulations (EC) No 1083/2006 and No 1198/2006 and Commission Regulations (EC) No 1828/2006 and No 498/2007 for substantive tests (audit of operations). Non-statistical selection should only be used in extreme cases where statistical selection is impossible, e.g. associated to very small populations or sample sizes (see Section 6.2).

5.2 Selection method

The selection method can belong to one of two broad categories:

- Statistical selection, or
- Non-statistical selection.

Statistical selection includes two possible techniques:

- Random selection;
- Systematic selection.

In random selection random, numbers are generated for each population unit in order to select the units constituting the sample.

Systematic sampling uses a random starting point and then applies a systematic rule to select the additional items (e.g. each 20th item after the random starting point).

Usually the equal probability methods are based on random selection and MUS is based on systematic selection.

Non-statistical selection covers the following possibilities (among others):

- Haphazard selection
- Block selection
- Judgement selection
- Risk based sampling combining elements of the three possibilities above

Haphazard selection is “false random” selection, in the sense of an individual “randomly” selecting the items, implying an unmeasured bias in the selection (e.g. items easier to analyse, items easily assessed, items picked from a list displayed particularly on the screen, etc…).

Block selection is similar to cluster sampling (as of groups of population units), where the cluster is picked non-randomly.

Judgment selection is purely based on the auditor’s discretion, whatever the rationale (e.g. items with similar names, all operations related to a specific domain of research, etc…).

Risk-based sampling is a non-statistical selection of items based on various intentional elements, often taking from all three non-statistical selection methods.

5.3 Projection (estimation)

As stated before the final goal when applying a sampling method is to project (extrapolate or estimate) the level of error (misstatement) observed in the sample to the whole population. This process will allow to conclude whether a population is materially misstated or not and, if so, by how much (an error amount). Therefore, the level of error found in the sample is not of interest by itself, being merely instrumental, i.e. a mean through which the error is projected to the population.

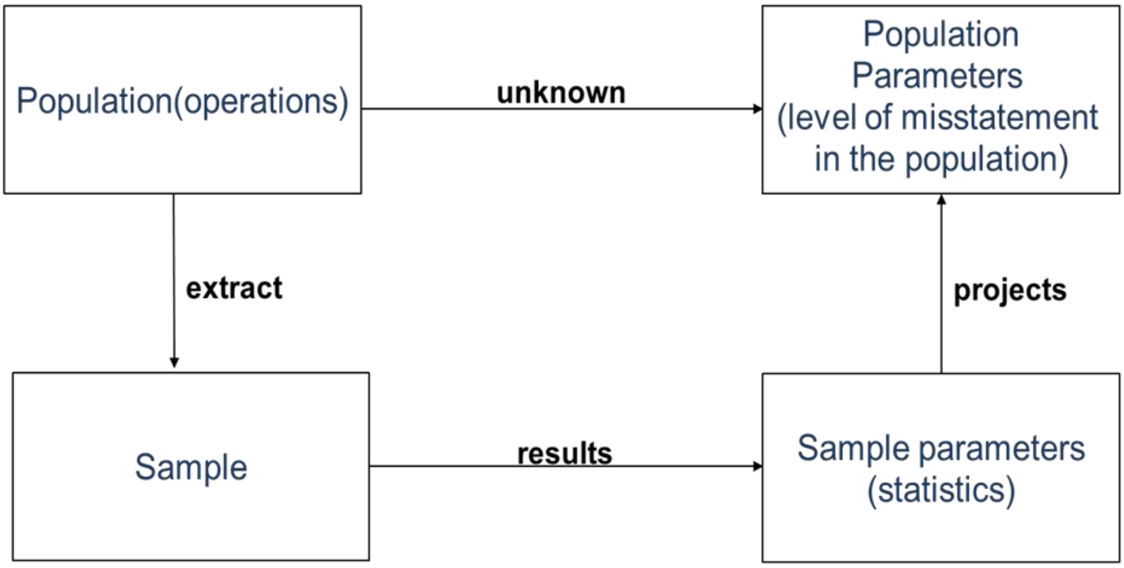


Fig. 3 Sample selection and projection

Sample statistics used to project the error to the population are called estimators. The act of projection is called estimation and the value calculated from the sample (projected value) is called the estimate. Clearly, this estimate, only based in a fraction of the population is affected by an error called the sampling error.

5.4 Precision (sampling error)

This is the error that arises because we are not observing the whole population. In fact, sampling always implies an estimation (extrapolation) error as we rely on sample data to extrapolate to the whole population. Sampling error is an indication of the difference between the sample projection (estimate) and the true (unknown) population parameter (value of error). It represents in fact the uncertainty in the projection of results to the population. A measure of this error is usually called **precision** or accuracy of the estimation. It depends mainly on **sample size**, **population variability** and in smaller degree **population size**.

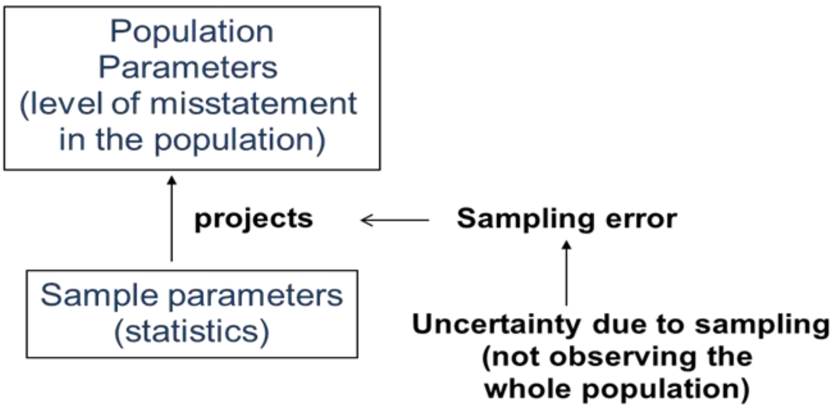


Fig. 4 Sampling error

A distinction should be made between planned precision and effective precision (SE in the formulas presented in Section 7). While planned precision is the maximum planned sampling error for sample size determination (usually is the difference between maximum tolerable error and the anticipated error and it should be set to a value lower than the materiality level), the effective precision is an indication of the difference between the sample projection (estimate) and the true (unknown) population parameter (value of error) and represents the uncertainty in the projection of results to the population.

5.5 Population

The population for sampling purposes includes the expenditure certified to the Commission for operations within a programme or group of programmes in the reference year. All operations, for which declared expenditure has been included in certified statements of expenditure submitted to the Commission during the year subject to sample, should be comprised in the sampled population, except when the population of operations is too small for statistical sampling (see below section 5.7)

In general, all the expenditure declared to the Commission for all the selected operations in the sample should be subject to audit. Nevertheless, whenever the selected operations include a large number of **payment claims** or invoices they can be audited through sampling, selecting the claims/invoices by using the same principles used to select the operations. In this case, appropriate sample sizes should be calculated within each operation[7]. Whenever this approach is followed, the sampling methodology should be recorded in the audit report or working papers.

It can happen that, when breaking down the expenditure certified by payment claims submitted to the Certifying Authority, there are some claims with **negative amounts** corresponding to corrections done by Managing Authority. In this case or similar ones, the negative amounts should constitute a separate population and should be audited separately[8] since the objective in this case will be to verify if the amount corrected corresponds to what has been decided by the Member State or the Commission. In case the AA concludes that the amount corrected is less than what was decided, then this matter should be disclosed in the Annual Control Report under section "8 - other information", in particular when this non-compliance constitutes an indication of weaknesses in the Member State's corrective capacity set out in Article 61(f) of Regulation (EC) No 1083/2006. In this context, when calculating the projected error, this only concerns the errors found in the population of positive amounts and this is the book value to be considered in the total projected error rate.

[7] This approach corresponds to a two-stage sampling design. The exact determination of sample size for two stage sampling is out of the scope of these notes. Despite the methodology used to determine sample sizes in statistical sampling, a basic rule of thumb is to never use sample sizes smaller than 30 observations.

[8] Of course, the AA may also draw a sample of this separate population.

Before calculating the projected error rate, the AA should verify that the errors found are not already corrected in the reference year (i.e. included in the population of negative amounts, as described above). If this is the case, these errors should not be included in the projected error rate.

The audit authority may decide to widen the audit to other related expenditure declared by the selected operations outside the reference period, in order to increase the efficiency of the audits. The results from checking additional expenditure outside the reference period should not be taken into account for determining the total projected error rate.

5.6 Stratification

We talk about stratification whenever the population is divided in sub-populations called strata and independent samples are drawn from each stratum.

The main goal of stratification is two-folded: on one hand usually allows an improvement of precision (for the same sample size) or a reduction of sample size (for the same level of precision); on the other hand assures that the subpopulations corresponding to each stratum are represented in the sample.

Whenever we expect that the level of error (misstatement) will be different for different groups in the population (e.g. by programme, region, intermediate body, risk of the operation) this classification is a good candidate to implement stratification. The stratification by level of expenditure per operation is also possible and desired, mainly when used in combination with the equal probability sampling methods.

Different sampling methods can be applied to different strata. For example, it is common to apply a 100% audit of the high-value items and apply a statistical sampling method to audit a sample of the remaining lower-value items that are included in the additional stratum or strata. This is useful in the event that the population include a few quite high-value items, as it lowers the variability in each stratum and therefore allows an improvement of precision (or reduction of sample size).

5.7 Sampling unit

The unit to be selected for audit is generally the operation. Where an operation consists of a number of distinct projects, they may be identified separately for sampling purposes. When the population of operations is too small for statistical sampling (i.e. between 50 and 150 population units), the unit to be selected for audit may be the beneficiary's payment claim.

5.8 Materiality

As follows from annex IV of the Regulations, a materiality level of 2% maximum is applicable to the expenditure declared to the Commission in the reference year. The AA can consider reducing the materiality for planning purposes (tolerable error). The materiality is used:

- As a threshold to compare the projected error in expenditure
- To define the tolerable/acceptable error that is used for determining sample size

5.9 Tolerable error and planned precision

The tolerable error is the maximum acceptable error rate that can be found in the population for a certain year. With a 2% materiality level this maximum tolerable error is therefore 2% of the expenditure certified to the Commission for that reference year.

The planned precision is the maximum sampling error accepted for the projection of errors in a certain year, i.e. the maximum deviation between the true population error and the projection produced from sample data. It should be set by the auditor to a value lower the tolerable error, because otherwise the results of sampling of operations will have a high risk of being inconclusive and a complementary sample may be needed.

For example, for a population with total book value of 10,000,000 € the corresponding tolerable error is 200,000 € (2% of the total book value). If the projected error is 5,000€ and the auditor sets the precision exactly to 200,000 € (this error arises because the auditor is only observing a small part of the population, i.e. the sample), then the upper error limit (upper limit of the confidence interval) will be about 205,000€. This is an inconclusive result as we have a very small projected error but an upper limit that exceeds the materiality threshold.

The most adequate way to settle the planned precision is to calculate it equal to the difference between the tolerable error and the anticipated error (the projected error that the auditor expects to obtain at the end of the audit). This anticipated error will of course be based on the auditor professional judgment, supported by the evidence gathered in the auditing activities in previous years for the same of similar population or in preliminary/pilot sample.

Note that the choice of a realistic anticipated error is important, since the sample size is highly dependent on the value chosen for this error. See also section 8.1.

Section 7 presents detailed formulas to use in the sample size determination process.

5.10 Variability

The variability of the population is a very influential parameter on sample size. Variability is usually measured by a parameter known as standard-deviation[9] and usually represented by σ. For example, for a population of 100 operations where all operations have the same level of error of € 1,000,000 € (average error of μ = 1,000,000 €) there is no variability (indeed, the standard-deviation of errors is zero).On the other hand, for a population of 100 operation in which 50 share an error of 0€ and the remaining 50 share an error of 2,000,000 € (the same average error of μ = 1,000,000 €) the standard-deviation of errors is high (1,000,000€).

The sample size needed to audit a population of low variability is smaller than the one needed for a population of high variability. In the extreme case of the first example (with a variance of 0), a sample size of one operation would be sufficient to project the population error accurately.

The standard-deviation is the most common measure of variability as it is more easily understandable than variance. Indeed the standard-deviation is expressed in the units of the variable which variability it seeks to measure. On the contrary, the variance is expressed in the square of the units of the variable which variability it measures and it is a simple average of the squares of the variable deviance values around the mean:

$$s^2 = \frac{1}{\#\,of\ units} \sum_{i=1}^{\#\,of\ units} (V_i - \bar{V})^2$$

where V_i represents the individual values of the variable V and $\bar{V} = \frac{\sum_{i=1}^{\#\,of\ units} V_i}{\#\,of\ units}$ represents the mean error. The standard-deviation is simply the square-root of the variance:

$$s = \sqrt{s^2}$$

The standard deviation of the errors of the examples mentioned at the beginning of this section can be calculated as:

a) Case 1
    a. N=100
    b. All the operation have the same level of error of € 1,000,000 €
    c. Mean error

[9] The standard deviation is a measure of the variability of the population around its mean. It can be calculated using errors or book-values. When calculated over the population is usually represented by σ and when calculated over the sample is represented by s. The larger the standard deviation the more heterogeneous is the population (or the sample). The variance is the square of the standard deviation.

$$\frac{\sum_{i=1}^{100} 1{,}000{,}000}{1000} = \frac{100 \times 1{,}000{,}000}{100} = 1{,}000{,}000$$

d. Standard deviation of errors

$$s = \sqrt{\frac{1}{100}\sum_{i=1}^{100}(1{,}000{,}000 - 1{,}000{,}000)^2} = 0$$

b) Case 2

a. N=100

b. 50 operations have 0 of error and 50 operations have 2,000,000 € of error

c. Mean error

$$\frac{\sum_{i=1}^{50} 0 + \sum_{i=1}^{50} 2{,}000{,}000}{1000} = \frac{50 \times 2{,}000{,}000}{100} = 1{,}000{,}000$$

d. Standard deviation of errors

$$s = \sqrt{\frac{1}{100}\left(\sum_{i=1}^{50}(0 - 1{,}000{,}000)^2 + \sum_{i=1}^{50}(2{,}000{,}000 - 1{,}000{,}000)^2\right)}$$

$$= \sqrt{\frac{50 \times 1{,}000{,}000^2 + 50 \times 1{,}000{,}000^2}{100}}$$

$$= \sqrt{1{,}000{,}000^2} = 1{,}000{,}000$$

5.11 Confidence interval and Upper Limit of Error

It is the interval that contains the true (unknown) population value (error) with a certain probability (called confidence level). It has the general form:

$$[EE - SE; EE + SE]$$

where

- EE represents the projected or extrapolated error; also corresponds to the Most Likely Error (MLE) in the MUS terminology;
- SE represents the precision (sampling error);

The projected/extrapolated error (EE) and the Upper Limit of Error (EE+SE) are the two most important instruments to conclude whether a population of operations is materially misstated or not. Of course, the ULE can only be calculated when statistical sampling is used; hence, for non-statistical sampling the EE is always the best estimate of the error in the population.

When statistical sampling is used, the following situations can arise:

- If EE is larger than the materiality threshold (hereafter 2%, for simplification) , then the AA concludes that there is material error;
- If EE is lower than 2% and the ULE is lower than 2%, the AA concludes that the population is not misstated by more than 2% at the specified level of sampling risk.
- If EE is lower than 2% but the ULE is larger than 2%, the AA concludes that additional work is needed. Accordingly to the INTOSAI guideline n° 23[10], the additional work can include:
  - *"requesting the audited entity to investigate the errors/exceptions found and the potential for further errors/exceptions. This may lead to agreed adjustments in the financial statements;*
  - *carrying out further testing with a view to reducing the sampling risk and thus the allowance that has to be built into the evaluation of results;*
  - *using alternative audit procedures to obtain additional assurance."*

The AA should use its professional judgment to select one of the options indicated above and report accordingly in the ACR.

Attention is drawn for the fact that, in most cases where an ULE is well above 2% this could be prevented or minimized if the AA considers a realistic anticipated error when calculating the original sample size (see section 8.2.2 below, for more details).

When following in the third option (projected error is lower than 2% but the ULE is higher than 2%), in some cases, the AA may find that the results are still conclusive for a smaller confidence level than the planned one. When this recalculated confidence level is still compatible with an assessment of the quality of the management and control systems, it will be perfectly safe to conclude that the population is not materially misstated even without carrying out additional audit work. Therefore, it is advisable to perform the recalculation of the confidence level and only in situations where the recalculated confidence is not acceptable (not in accordance with the assessment of the systems) proceed with the additional work suggested above. See Section 8.7 for an explanation of the recalculation of confidence levels.

5.12 Confidence level

The confidence level is set by the Regulation for the purpose of defining the sample size for substantive tests.

As the sample size is directly affected by the confidence level, the objective of the Regulation is clearly to offer the possibility of reducing audit workload for systems with an established low error rate (and therefore high assurance), while maintaining the

[10] See http://eca.europa.eu/portal/pls/portal/docs/1/133817.PDF

requirement to check a high number of items in the case a system has a potentially high error rate (and therefore low assurance).

The easiest way to interpret the meaning of confidence level is the probability that a confidence interval produced by sample data contains the true population error (unknown). For example, if the error in the population is projected to be 6,000,000€ and the 90% confidence level interval is

$$[5{,}000{,}000€; 7{,}000{,}000€],$$

it means that there is 90% probability of the true (but unknown) population error is between these two bounds. The implications of these strategic choices for the audit planning and sampling of operations are explained in the following chapters.

5.13 Error rate

The **sample error rate** is computed as the ratio between total error in the sample and total book value of the sampled items, the **projected error rate** is computed as the ratio between **projected population error** and total book value. Again, note that the sample error is of no interest by itself as it should be considered a mere instrument to calculate the projected error[11].

[11] In some sampling methods, namely the ones based on equal probability selection, the sample error rate can be used to project the population error rate.

6 Sampling techniques for the audit of operations

6.1 Overview

Within the audit of operations, the purpose of sampling is to select the operations to be audited through substantive tests; the population comprises the expenditure certified to the Commission for operations within a programme/group of programmes in the reference year.

Figure 5 shows a summary of the most used sampling methods for audit.

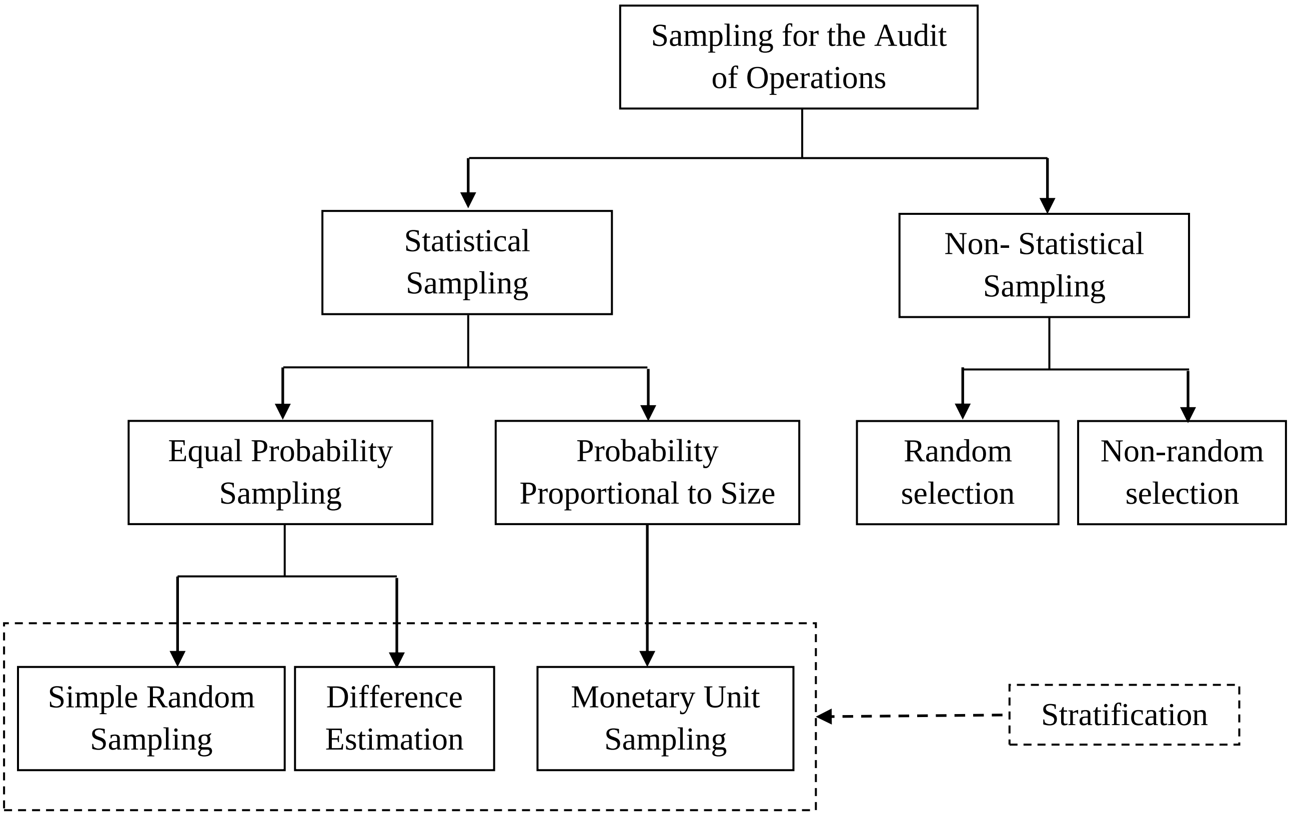


Fig. 5 Sampling methods for the audit of operations

As stated before, please note that the first distinction between sampling methods is made between statistical and non-statistical sampling.

Statistical sampling is required by Council Regulations (EC) No 1083/2006 and No 1198/2006 and Commission Regulations (EC) No 1828/2006 and No 498/2007 for substantive tests (audit of operations). Non-statistical selection should only be used in extreme cases where statistical selection is impossible, e.g. associated to very small populations or sample sizes. Section 6.2 discusses the conditions of applicability of the different sampling designs and refers the unique extreme situations where non-statistical sampling is admissible.

Within statistical sampling, the major distinction between methods is based on the selection probabilities: equal-selection probabilities methods (including simple random sampling and difference estimation) and probability proportional to size methods where the well-known monetary unit sampling (MUS) method stands out.

Monetary unit sampling (MUS) is in fact a probability-proportional-to-size (PPS). The name comes from the fact that operations are selected with probabilities proportional to their monetary value. The higher the monetary value the higher the probability of selection. Again, favourable conditions for the application of each specific method are discussed in the following section.

Despite the specific sampling method that is selected, auditing the operations through sampling should always follow a basic common structure:

1. **Define the objectives of the substantive tests**: usually the determination of the level of error in the expenditure certified to the Commission for a given year for a programme (or group of programmes) based on a projection from a sample.
2. **Define the population**: expenditure certified to the Commission for a given year for a programme or for a group of programmes, and the **sampling unit**, which is the item to be selected to the sample (usually the operation, but other possibilities are available as the payment claim).
3. **Define population parameters**: this included defining the tolerable error (2% of the expenditure certified to the Commission), the anticipated error (expected by the auditor), the confidence level (taking into account the audit risk model) and (usually) a measure of population variability.
4. **Determine the sample size**, according to the sampling method used. **<u>It is important to note that the final sample size is always rounded up to the nearest integer.</u>**
5. **Select the sample and perform the audit**.
6. **Project results, calculate precision and draw conclusion**: this step covers the computation of the precision and projected error and comparing these results with the materiality threshold.

The choice of a particular sampling method refines this archetypal structure, by providing a formula to compute the sample size and a framework for projecting results.

Also note that the specific formulas for sample size determination vary with the chosen sampling method. Nevertheless, despite the chosen method the sample size will always depend on three parameters:

- The confidence level (the higher the confidence level the larger the sample size)

- The variability of the population (i.e. how variable are the values of the population; if all the operations in the population have similar values of error the population is said to be less variable than a population where all the operations show extremely different values of error). The higher the variability of the population the larger the sample size.
- The planned precision set by the auditor; this planned precision is typically the difference between the tolerable error of 2% of the expenditure and the anticipated error. Assuming an anticipated error below 2%, the larger the anticipated error (or the smaller the planned precision) the larger the sample size.

The sample size also depends on population size but on a very lower degree. For reasonable large populations the sample size is almost independent of the population size. This means that in normal circumstances the sample size needed to represent a population of 10,000 operations is almost the same that is needed to assess a population of 20,000 operations. Note that, in extreme cases of very small populations, sometimes population size cannot be ignored.

Specific formulas for determining sample size are offered in Section 7. Nevertheless, one important rule of the thumb is never to use a sample size smaller than 30 units (in order that the distributional assumptions used to create confidence intervals will hold).

6.2 Conditions of applicability of sampling designs

As a preliminary remark on the choice of a method to select the operations to be audited, whilst the criteria that should lead to this decision are numerous, from a statistical point of view the choice is mainly based on the expectation regarding the variability of errors and their relationship with the expenditure.

The table below gives some indications on the most appropriate methods depending on the criteria.

Sampling Method	Favourable conditions
Standard MUS	Errors have high variability[12] and are approximately proportional to the level of expenditure (i.e. error rates are of low variability) The values of expenditure per operation show high variability
Conservative MUS	Errors have high variability and are approximately proportional to the level of expenditure The values of expenditure per operation show high variability Proportion of errors is expected to be low[13]
Difference estimation	Errors are relatively constant or of low variability An estimate of the total corrected expenditure in the population is needed
Simple random sampling	General proposed method that can be applied when the previous conditions do not hold Can be applied using mean-per-unit estimation or ratio estimation (see Section 7.1.1.3 for guidelines for choosing between these two estimation techniques)
Non-statistical methods	If the application of statistical method is impossible (see discussion below)
Stratification	Can be used in combination with any of the above methods It is particularly useful whenever the level of error is expected to vary significantly among population groups (subpopulations)

Table 3. Favourable conditions for the choice of sampling methods

Although the previous advices should be followed, actually no method can be universally classified as the only suited method or even the “best method”. In general, all methods can be applied. The consequence of choosing a method that is not the most suitable for a certain situation is that the sample size will have to be larger than the one obtained when using a more appropriate method. Nevertheless, it will always be possible to select a representative sample trough any of the methods, provided that an adequate sample size is considered.

Also note that stratification can be used in combination with any sampling method. The reasoning underlying stratification is the partition of the population in groups (strata)

[12] High variability means the errors across operations are not similar, that is, there are small and large errors in contrast with the case where all the errors are more or less of similar values (cf. section 5.10).
13As the MUS conservative approach is based on a distribution for rare events, is it particularly suited when the ratio of number of errors to the total number of operations in the population (proportion of errors) is expected to be low.

more homogeneous (with less variability) than the whole population. Instead of having a population with high variability it is possible to have two or more subpopulations with lower variability. Stratification should be used to either **minimise variability or isolate error-generating subsets of the population**. In both cases stratification will reduce the needed sample size.

As stated before, statistical sampling should be used to draw conclusions about the amount of error in a population. However, there are special cases when a statistical method cannot be applied.

In fact, non-statistical sampling should only be used only:

- when having an extremely small population, whose size won't support the selection of a sample of adequate size (the population is smaller or very close to the recommended sample size)[14].
- when it is not possible to observe the sample size that would be advisable for a statistical method, due to uncontrollable restrictions.

The audit authority must use all possible means to achieve a sufficiently large population: by grouping programs, when part of a common system; and/or by using as the unit the beneficiaries' periodic payment claims. AA should also consider that even in an extreme situation where the statistical approach is not possible in the beginning of the program period, it should be applied as soon as it is feasible.

6.3 European Territorial Cooperation (ETC) programmes

ETC programmes have a number of particularities: it will not normally be possible to group them because each programme system is different; the number of operations is frequently low; for each operation there is generally a lead partner and a number of other project partners.

The guidance set out above for the case of programmes with a small number of operations should be followed, taking into account the following additional procedures.

Firstly, in order to obtain a sufficiently large population for the use of a statistical sampling method, it may be possible to use as sampling unit the underlying validated payment claims of each partner beneficiary in an operation. In this case the audit will be carried out at the level of each beneficiary selected, and not necessarily the lead partner of the operation.

In case a sufficiently large population cannot be obtained to carry out statistical sampling, Option 1 or Option 2 mentioned in Section 7.4.1 should be applied.

[14] Cf. section 7.4.1.

For the operations selected, the audit of the lead partners should always be carried out covering both its own expenditure and the process for aggregating the project partners' payment claims. Where the number of project partners is such that it is not possible to audit all of them, a random sample can be selected. The size of the combined sample of lead partner and project partners must be sufficient to enable the audit authority to draw valid conclusions.

6.4 Notation

Before presenting the main sampling methods for audit of operations it is useful to define a set of concepts related to sampling that are common to all the methods. Thus:

- z is a parameter from the normal distribution related to the confidence level determined from system audits. The possible values of z are presented in the following table. A complete table with values of the normal distribution can be found in appendix.

Confidence level	**60%**	**70%**	**80%**	**90%**	**95%**
System assurance level	High	Moderate	Moderate	Low	No assurance
z	0.842	1.036	1.282	1.645	1.960

Table 4. Values of z by confidence level

- N is the population size (e.g. number of operations in a programme or payment claims); if the population is stratified, an index h is used to denote the respective stratum, $N_h, h = 1,2,\ldots,H$ and H is the number of strata;
- n is the sample size; if the population is stratified, an index h is used to denote the respective stratum, $n_h, h = 1,2,\ldots,H$ and H is the number of strata;
- TE be the maximum tolerable error admissible by the regulation, that is, 2% of the total expenditure certified to the Commission (the Book Value, BV);
- $BV_i, i = 1,2,\ldots,N$ is the book value (the expenditure certified to the Commission) of an item (operation/payment claim);
- $CBV_i, i = 1,2,\ldots,N$ is the corrected book value, the expenditure determined after auditing procedures of an item (operation/payment claim);
- $E_i = BV_i - CBV_i, i = 1,2,\ldots,N$, is the amount of error of an item and is defined as the difference between the book value of the i-th item included in sample and the respective corrected book value; if the population is stratified an index h is used to denote the respective stratum, $E_{hi} = BV_{hi} - CBV_{hi}, i = 1,2,\ldots,N_h, h = 1,2,\ldots,H$ and H is the number of strata;
- AE is the anticipated error defined by the auditor based on the expected level of error at the level of the operations (e.g. an anticipated error rate times the Total

expenditure at the level of the population). AE can be obtained from historical data (projected error in past period) or from a preliminary/pilot sample of low sample size (the same used to determine the standard deviation).

7 Sampling methods

7.1 Simple random sampling

7.1.1 Standard approach

7.1.1.1 Introduction

Simple random sampling is a statistical sampling method. It is the most well-known among the equal probability selection methods. Aims to project to the level of error observed in the sample to the whole population.

The statistical unit to be sampled is the operation (or payment claim). Units in the sample are selected randomly with equal probabilities. Simple random sampling is a generic method that fits every kind of population, although, as it does not use auxiliary information, usually requires larger sample sizes than MUS (whenever the level of expenditure varies significantly among operations and there is positive association between expenditure and errors). The projection of errors can be based on two sub-methods: mean-per-unit estimation or ratio estimation (see Section 7.1.1.3).

As all other methods, this method can be combined with stratification (favourable conditions for stratification are discussed in Section 6.2 and specific formulas are presented in Section 7.1.2)

7.1.1.2 Sample size

Computing sample size n within the framework of simple random sampling relies on the following information:

- Population size N
- Confidence level determined from systems audit and the related coefficient z from a normal distribution (see Section 6.4)
- Maximum tolerable error TE (usually 2% of the total expenditure)
- Anticipated error AE chosen by the auditor according to professional judgment and previous information
- The standard deviation σ_e of the errors.

The sample size is computed as follows[15]:

[15] When dealing with a small population size, i.e. if the final sample size represents a large proportion of the population (as a rule of thumb more than 10% of the population) a more exact formula can be used leading to $n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2 \Big/ \left(1 + \left(\frac{\sqrt{N} \times z \times \sigma_e}{TE - AE}\right)^2\right)$. This correction is valid for simple random sampling and for difference estimation. It can also be introduced in two steps by calculating the sample size n with the usual formula and sequentially correct it using $n' = \frac{n \times N}{n + N - 1}$.

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

where σ_e is the standard-deviation of errors in the population. Note that this standard-deviation of the errors for the total population is assumed to be known in the above calculation. In practice, this will almost never be the case and Member-States will have to rely either on historical data (standard-deviation of the errors for the population in the past period) or on a preliminary/pilot sample of low sample size (sample size is recommended to be not smaller than 20 to 30 units). In the latter case a preliminary sample of size n^p is selected and a preliminary estimate of the variance of errors (square of the standard-deviation) is obtained though

$$\sigma_e^2 = \frac{1}{n^p - 1}\sum_{i=1}^{n^p}(E_i - \bar{E})^2,$$

where E_i represent the individual errors for units in the sample and $\bar{E} = \frac{\sum_{i=1}^{n^p} E_i}{n^p}$ represents the mean error of the sample.

Note that the pilot sample can subsequently be used as a part of the sample chosen for audit.

7.1.1.3 Projected error

There are two possible ways to project the sampling error to the population. The first is based on mean-per-unit estimation (absolute errors) and the second on ratio estimation (error rates).

Mean-per-unit estimation (absolute errors)
Multiply the average error per operation observed in the sample by the number of operations in the population, yielding the projected error:

$$EE_1 = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$

Ratio estimation (error rates)
Multiply the average error rate observed in the sample by the book value at the level of the population:

$$EE_2 = BV \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV_i}$$

The sample error rate in the above formula is just the division of the total amount of error in the sample by the total amount of expenditure of units in the sample (expenditure audited).

It is not possible to know a priory which is the best extrapolation method as their relative merits depend on the level of association between errors and expenditure. As a basic rule of thumb, the second method should just be used when there is the expectation of high association between errors and expenditure (higher value items tend to exhibit higher errors) and the first method (Mean per unit) when there is an expectation that errors are relatively independent from the level of expenditure (higher errors can be found either in units of high or low level of expenditure)[16]. This assessment can be made using sample data as the decision about the extrapolation method can be taken after the sample is selected and audited.

7.1.1.4 Precision

Remember that precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). It is calculated differently according to the method that has been used for extrapolation.

Mean-per-unit estimation (absolute errors)
The precision is given by the following formula

$$SE_1 = N \times z \times \frac{s_e}{\sqrt{n}}$$

where s_e is the standard-deviation of errors in the sample (now calculated from the same sample used to project the errors to the population)

$$s_e^2 = \frac{1}{n-1} \sum_{i=1}^{n} (E_i - \bar{E})^2$$

16 Exact formulas to determine the best method are out of the scope of these notes. Formally the second method tends to be better whenever $\frac{\mathrm{COV_{E,BV}}}{\mathrm{VAR_{BV}}} > R/2$, where $\mathrm{COV_{E,BV}}$ represents the covariance between errors and expenditure, $\mathrm{VAR_{BV}}$ the variance of expenditure and R the error rate in the population.

Ratio estimation (error rates)
The precision is given by the following formula

$$SE_2 = N \times z \times \frac{s_q}{\sqrt{n}}$$

where s_q is the sample standard deviation of the variable q:

$$q_i = E_i - \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV_i} \times BV_i.$$

This variable is for each unit in the sample computed as the difference between its error and the product between its book value and the error rate in the sample.

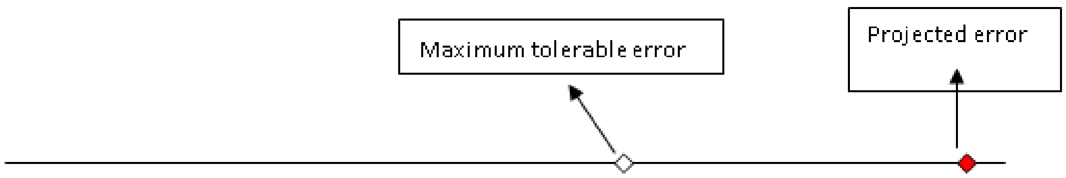
7.1.1.5 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

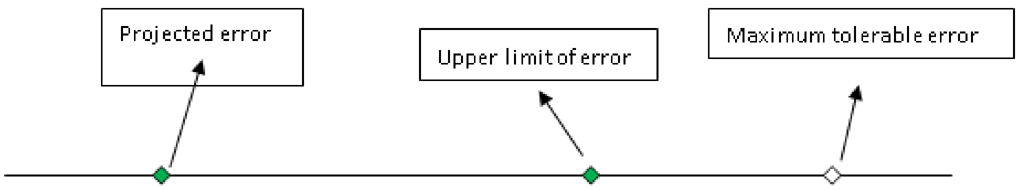
$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

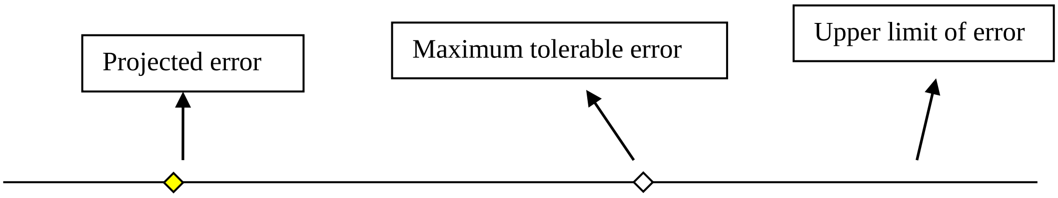
- If projected error is larger than maximum tolerable error, it means that the auditor would conclude that there is enough evidence to support that errors in the population are larger than materiality threshold:



- If the upper limit of error is lower than maximum tolerable error, then the auditor should conclude that errors in the population are lower than materiality threshold.



- If the projected error is lower than maximum tolerable error but the upper limit of error is larger than the maximum tolerable error, this means that additional work is needed as there is not enough evidence to support that the population is not materially misstated. The specific additional work needed is discussed in Section 5.11.



7.1.1.6 Example

Let us assume a population of expenditure certified to the Commission in a given year for operations in a programme or group of programmes. The system audits carried out by the audit authority have yielded a moderate assurance level. Therefore, a confidence level of 80% seems to be adequate for audit of operations.

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference year)	3,199,654,543 €

A preliminary sample of 20 operations yielded a preliminary estimate for the standard deviation of errors of 62,194 € (computed in MS Excel as **":=STDEV(D2:D21)**"):

	A	B	C	D
1	**Operation**	**Book Value (BV)**	**Audited Value (AV)**	**Error**
2	2,299	1,368,071 €	1,368,071 €	- €
3	2,579	2,324,672 €	2,267,903 €	56,768 €
4	3,736	4,521,748 €	4,245,900 €	275,848 €
5	4,068	11,905,284 €	11,905,284 €	- €
6	2,020	1,237,076 €	1,237,076 €	- €
7	13	850,647 €	850,647 €	- €
8	2,761	987,452 €	987,452 €	- €
9	3,384	6,863,730 €	6,863,730 €	- €
10	4,266	737,302 €	737,302 €	- €
11	1,273	1,748,231 €	1,748,231 €	- €
12	1,506	1,426,326 €	1,426,326 €	- €
13	1,882	885,025 €	885,025 €	- €
14	143	721,564 €	705,720 €	15,845 €
15	2,246	3,398,994 €	3,398,994 €	- €
16	2,841	1,913,322 €	1,913,322 €	- €
17	1,757	845,404 €	845,404 €	- €
18	4,998	3,701,924 €	3,701,924 €	- €
19	4,828	1,633,711 €	1,633,711 €	- €
20	646	1,981,079 €	1,981,079 €	- €
21	2,635 €	975,033 €	975,033 €	- €
22	**Total**	50,026,595 €	49,678,134 €	348,461 €
23	**Sample error rate:=D22/B22**			0.7%
24	**Sample standard deviation of errors:=STDEV(D2:D21)----->**			62,194 €

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

where z is 1.282 (coefficient corresponding to a 80% confidence level), σ_e is 62,194 € and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value, i.e. 2% x 3,199,654,543 € = 63,993,091 €. This preliminary sample yields a sample error rate of 0.7%. Further, based either on previous year experience and on the conclusions of the report on managing and control systems the audit authority expects a error rate not larger than 0.7%, Thus AE, the anticipated error, is 0.6% of the total expenditure, i.e., 0.7% x 3,199,654,543 € = 22,397,582 €:

$$n = \left(\frac{3{,}852 \times 1.282 \times 62{,}194}{63{,}993{,}091 - 22{,}397{,}582}\right)^2 \approx 55$$

The minimum sample size is therefore 55 operations.

The previous preliminary sample of 20 is used as part of the main sample. Therefore, the auditor only has to randomly select 35 further operations. The following table shows the results for the whole sample of 55 operations:

	A	B	C	D	E	F
1	**Operation ID (1)**	**Book Value (BV) (2)**	**Audited Value (AV) (3)**	**Error (4)**	**Error rate (4)/(2)**	**q (4)-SUM(4)/SUM(2)*(2)**
2	2,299	1,368,071 €	1,368,071 €	- €	0.0%	- 7,016 €
3	2,579	2,324,672 €	2,267,903 €	56,768 €	2.4%	44,847 €
4	3,736	4,521,748 €	4,245,900 €	275,848 €	0.7%	252,660 €
5	4,068	11,905,284 €	11,905,284 €	- €	0.0%	- 61,053 €
6	2,020	1,237,076 €	1,237,076 €	- €	0.0%	- 6,344 €
7	13	850,647 €	850,647 €	- €	0.0%	- 4,362 €
8	2,761	987,452 €	987,452 €	- €	0.0%	- 5,064 €
9	3,384	6,863,730 €	6,863,730 €	- €	0.0%	- 35,199 €
10	4,266	737,302 €	737,302 €	- €	0.0%	- 3,781 €
11	1,273	1,748,231 €	1,748,231 €	- €	0.0%	- 8,965 €
12	1,506	1,426,326 €	1,426,326 €	- €	0.0%	- 7,315 €
13	1,882	885,025 €	885,025 €	- €	0.0%	- 4,539 €
14	143	721,564 €	705,720 €	15,845 €	2.2%	12,144 €
15	2,246	3,398,994 €	3,398,994 €	- €	0.0%	- 17,431 €
16	2,841	1,913,322 €	1,913,322 €	- €	0.0%	- 9,812 €
17	1,757	845,404 €	845,404 €	- €	0.0%	- 4,335 €
18	4,998	3,701,924 €	3,701,924 €	- €	2.5%	- 18,984 €
19	4,828	1,633,711 €	1,633,711 €	- €	0.0%	- 8,378 €
20	646	1,981,079 €	1,981,079 €	- €	0.0%	- 10,160 €
21	2,635	975,033 €	975,033 €	- €		
22	**(...)**	**(...)**	**(...)**	**(...)**	**(...)**	**(...)**
56	3,873	470,763 €	470,763 €	- €	0.0%	- 2,354 €
57	**Total**	**52,056,328 €**	**51,509,453 €**	**546,875 €**		
58	**Sample standard deviation:=STDEV(D2:D56)-->**			56,539 €		58,412 €

The total book value of the 55 sampled operations is 52,056,328 € (computed in MS Excel as "**:=SUM(B2:B56)**"). The total error amount in the sample is 546,875 €

(computed in MS Excel as **":=SUM(D2:D56)**"). This amount, divided by the sample size, is the average operation error within the sample.

If we use mean-per-unit estimation, the projection of the error to the population is calculated by multiplying this average error by the population size (3,852 in this example). This figure is the projected error at the level of the programme:

$$EE_1 = N \times \frac{\sum_{i=1}^{55} E_i}{n} = 3{,}852 \times \frac{546{,}875}{55} = 38{,}301{,}136.$$

If we use ratio estimation, the projection of the errors to the population can be achieved by multiplying the average error rate observed in the sample by the book value at the level of the population:

$$EE_2 = BV \times \frac{\sum_{i=1}^{55} E_i}{\sum_{i=1}^{55} BV_i} = 3{,}199{,}654{,}543 \ \times \frac{546{,}875}{52{,}056{,}328} = 33{,}613{,}802$$

The sample error rate in the above formula is just the division of the total amount of error in the sample by the total amount of expenditure of operations in the sample.

The projected error rate is computed as the ratio between the projected error and the book value of the population (total expenditure). Using the mean-per-unit estimation the projected error rate is:

$$r_1 = \frac{38{,}301{,}136}{3{,}199{,}654{,}543} = 1.20\%$$

and using the ratio estimation is:

$$r_2 = \frac{33{,}613{,}802}{3{,}199{,}654{,}543} = 1.05\%$$

In both cases the projected error is smaller than the materiality level. However, final conclusions can only be drawn after taking into account the sampling error (precision).

The first step to obtain the precision is to calculate the standard deviation of errors in the sample (computed in MS Excel as **":=STDEV(D2:D56)**"):

$$s_e = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(E_i - \bar{E})^2} = \sqrt{\frac{1}{54}\sum_{i=1}^{55}(E_i - \bar{E})^2} = 56{,}539.$$

Thus, the precision of the mean-per-unit estimation is given by

$$SE_1 = N \times z \times \frac{s_e}{\sqrt{n}} = 3{,}852 \times 1.282 \times \frac{56{,}539}{\sqrt{55}} = 37{,}647{,}928.$$

For the ratio estimation it is necessary to create the variable

$$q_i = E_i - \frac{\sum_{i=1}^{55} E_i}{\sum_{i=1}^{55} BV_i} \times BV_i.$$

This variable is in the last column of the table (column F). For instance the value in cell F2 is given by the value of the error of the first operation (0 €) minus the sum of sample errors, in column D, 546,875 € ("**:=SUM(D2:D56)**") divided by the sum of sample book values, in column B, 52,056,328 € ("**:=SUM(B2:B56)**") and multiplied by the book value of the operation (768,071 €):

$$q_1 = 0 - \frac{546{,}875}{52{,}056{,}328} \times 1{,}368{,}071 = -7{,}016.$$

Given the standard deviation of this variable, $s_q = 58{,}412$ (computed in MS Excel as **":=STDEV(F2:F56)**"), the precision for ration estimation is given by the following formula

$$SE_2 = N \times z \times \frac{s_q}{\sqrt{n}} = 3{,}852 \times 1.282 \times \frac{58{,}412}{\sqrt{55}} = 38{,}895{,}113$$

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the projection

$$ULE = EE + SE$$

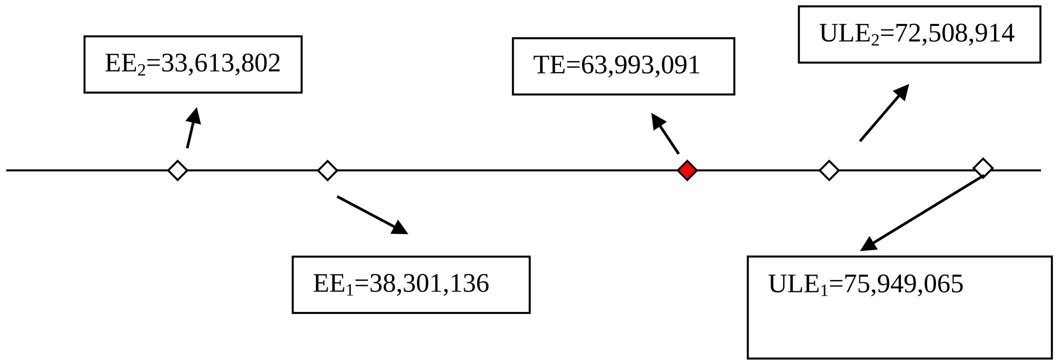
Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

$$ULE_1 = EE_1 + SE_1 = 38{,}301{,}136 + 37{,}647{,}928 = 75{,}949{,}065$$

or

$$ULE_2 = EE_2 + SE_2 = 33{,}613{,}802 + 38{,}895{,}113 = 72{,}508{,}914$$

Finally, comparing to the materiality threshold of 2% of the total book value of the programme (2% x 3,199,654,543 € = 63,993,091 €) with the projected error and upper limit of error, the conclusion is, using both approaches (mean-per-unit and ratio estimation), the projected error is lower than the maximum tolerable error, but the upper limit of error is larger the maximum tolerable error. The auditor is able to conclude that additional work is needed, as there is not enough evidence to support that the population is not materially misstated. The specific additional work needed is discussed in Section 5.11.



7.1.2 Stratified simple random sampling

7.1.2.1 Introduction

In stratified simple random sampling, the population is divided in sub-populations called strata and independent samples are drawn from each stratum, using the standard simple random sampling approach.

Candidate criteria to implement stratification should take into account that in stratification we aim to find groups (strata) with less variability than the whole population. With simple random sampling, the stratification by level of expenditure per operation is usually a good approach, whenever it is expected that the level of error is associated with the level of expenditure. Other variables that we expect to explain the level of error in the operations are also good candidates for stratification. Some possible choices are programmes, regions, intermediate bodies, classes based on the risk of the operation, etc.

If stratification by level of expenditure is implemented, consider to identify a high-value stratum[17], apply a 100% audit of these items, and apply simple random sampling to audit samples of the remaining lower-value items that are included in the additional stratum or strata. This is useful in the event that the population included a few high-value items. In this case, the items belonging to the 100% stratum should be taken out

[17] There is not a general rule to identify the cut-off value for the high value stratum. A rule of thumb would be to include all operations whose expenditure is larger than the materiality (2%) times the total population expenditure. More conservative approaches use a smaller cut-off usually dividing the materiality by 2 or 3, but the cut-off value depends on the characteristics of the population and should be based on professional judgment.

of the population and all the steps considered in the remaining sections will apply only to the population of the low-value items. Please note that it is not mandatory to audit 100% of the high-value stratum units. The AA may develop a strategy based on several strata, corresponding to different levels of expenditure, and have all the strata audited through sampling.

7.1.2.2 Sample size

The sample size is computed as follows

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors for the whole set of strata:

$$\sigma_w^2 = \sum_{i=1}^{H} \frac{N_h}{N} \sigma_{eh}^2, h = 1,2, \ldots, H;$$

and σ_{eh}^2 is the variance of errors in each stratum. The variance of the errors is computed for each stratum as an independent population as

$$\sigma_{eh}^2 = \frac{1}{n_h^p - 1} \sum_{i=1}^{n_h^p} (E_{hi} - \bar{E}_h)^2 , h = 1,2, \ldots, H$$

where E_{hi} represent the individual errors for units in the sample of stratum h and $\bar{E}_h$ represent the mean error of the sample in stratum h.

These values can be based on historical knowledge or on a preliminary/pilot sample of low sample size as previously presented for the standard simple random sampling method. In this later case the pilot sample can as usual subsequently be used as a part of the sample chosen for audit. If no historical information is available in the beginning of a programming period and it is not possible to access a pilot sample, the sample size may be calculated with the standard approach (for the first year of the period). The data collected in the audit sample of this first year can be used to refine sample size computation in the following years. The price to pay for this lack of information is that the sample size, for the first year, will probably be larger than the one that would be needed if auxiliary information about strata were available.

Once the total sample size, n, is computed the allocation of the sample by stratum is as follows:

$$n_h = \frac{N_h}{N} \times n.$$

This is a general allocation method, usually known as proportional allocation. Many other allocation methods are available. A more tailored allocation may in some cases bring additional precision gains or reduction of sample size. The adequacy of other allocation methods to each specific population requires some technical knowledge in sampling theory.

7.1.2.3 Projected error

Based on H randomly selected samples of operations, where the size of each one has been computed according to the above formula, the projected error at the level of the population can be computed through the two usual methods: mean-per-unit estimation and ratio estimation.

Mean-per-unit estimation

In each group of the population (stratum) multiply the average error per operation observed in the sample by the number of operations in the stratum (N_h); then sum all the results obtained for each stratum, yielding the projected error:

$$EE_1 = \sum_{h=1}^{H} N_h \times \frac{\sum_{i=1}^{n_h} E_i}{n_h}.$$

Ratio estimation

In each group of the population (stratum) multiply the average error rate observed in the sample by the population book value at the level of the stratum (BV_h):

$$EE_2 = \sum_{h=1}^{H} BV_h \times \frac{\sum_{i=1}^{n_h} E_i}{\sum_{i=1}^{n_h} BV_i}$$

The sample error rate in each stratum is just the division of the total amount of error in the sample of stratum by the total amount of expenditure in the same sample.

The choice between the two methods should be based upon the considerations presented for the standard simple random sampling method.

If a 100% stratum has been considered and previously taken from the population then the total amount of error observed in that exhaustive stratum should be added to the above estimate (EE_1 or EE_2) in order to produce the final projection of the amount of error in the whole population.

7.1.2.4 Precision

As for the standard method, precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). It is calculated differently according to the method that has been used for extrapolation.

Mean-per-unit estimation (absolute errors)
The precision is given by the following formula

$$SE_1 = N \times z \times \frac{s_w}{\sqrt{n}},$$

where s_w^2 is the weighted mean of the variance of errors for the whole set of strata (now calculated from the same sample used to project the errors to the population):

$$s_w^2 = \sum_{i=1}^{H} \frac{N_h}{N} s_{eh}^2, h = 1,2, \dots, H;$$

and s_{eh}^2 is the estimated variance of errors for the sample of stratum h

$$s_{eh}^2 = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} (E_{hi} - \bar{E}_h)^2 \, , h = 1,2, \dots, H$$

Ratio estimation (error rates)
The precision is given by the following formula

$$SE_2 = N \times z \times \frac{s_{qw}}{\sqrt{n}}$$

where

$$s_{qw}^2 = \sum_{h=1}^{H} \frac{N_h}{N} s_{qh}^2$$

is a weighted mean of the sample variances of the variable q_h, with

$$q_{ih} = E_{ih} - \frac{\sum_{i=1}^{n_h} E_{ih}}{\sum_{i=1}^{n_h} BV_{ih}} \times BV_{ih}.$$

This variable is for each unit in the sample computed as the difference between its error and the product between its book value and the error rate in the sample.

7.1.2.5 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 7.1.1.5.

7.1.2.6 Example

Let us assume a population of expenditure certified to the Commission in a given year for operations in a group of programmes. The management and control system is common to the group of programmes and the system audits carried out by the Audit Authority have yielded a moderate assurance level. Therefore, the audit authority decided to carry out audits of operation using a confidence level of 80%.

The AA has reasons to believe that there are substantial risks of error for high value operations, whatever the programme they belong to. Further, there are reasons to expect that there are different error rates across the programmes. Bearing in mind all this information, the AA decides to stratify the population by programme and by expenditure (isolating in a 100% sampling stratum all the operations with book value larger than the materiality).

The following table summarizes the available information.

Population size (number of operations)	3,852
Population size – stratum 1 (number of operations in programme 1)	2,520
Population size – stratum 2 (number of operations in programme 2)	1,327
Population size – stratum 3 (number of operations with BV > materiality level)	5
Book value (sum of the expenditure in the reference year)	4,199,882,024 €
Book value – stratum 1 (total expenditure in programme 1)	2,168,367,291 €
Book value – stratum 2 (total expenditure in programme 2)	1,447,155,510 €
Book value – stratum 3 (total expenditure of operations with BV > Materiality level)	584,359,223 €

The 100% sampling stratum containing the 5 high-value operation should be treated separately as stated in section 7.1.2.1. Therefore, hereafter, the value of N corresponds to the total number of operations in the population, deducted of the number of the operations included in the 100% sampling stratum, i.e. 3,847 (= 3,852 – 5) operations.

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where z is 1.282 (coefficient corresponding to a 80% confidence level) and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value, i.e. 2% x 4,199,882,024 € = 83,997,640 €. Based either on previous year experience and on the conclusions of the report on managing and control systems the audit authority expects an error rate not larger than 0.4%, Thus, AE, the anticipated error, is 0.4% of the total expenditure, i.e., 0.4% x 4,199,882,024 € = 16,799,528 €.

Since the third stratum is a 100% sampling stratum, the sample size for this stratum is fixed and is equal to the size of the population, that is, the 5 high-value operations. The sample size for the remaining two strata is computed using the above formula, where σ_w^2 is the weighted average of the variances of the errors for the two remaining strata:

$$\sigma_w^2 = \sum_{i=1}^{2} \frac{N_h}{N} \sigma_{eh}^2, h = 1{,}2;$$

and σ_{eh}^2 is the variance of errors in each stratum. The variance of the errors is computed for each stratum as an independent population as

$$\sigma_{eh}^2 = \frac{1}{n_h^p - 1} \sum_{i=1}^{n_h^p} (E_{hi} - \bar{E}_h)^2 \text{ , } h = 1,2, \dots, H$$

where E_{hi} represents the individual errors for units in the sample of stratum h and $\bar{E}_h$ represents the mean error of the sample in stratum h.

A preliminary sample of 20 operations of stratum 1 yielded an estimate for the standard deviation of errors of 5,370 €:

	A	B	C	D
1	**Operation ID**	**Book Value (BV)**	**Audited Value (AV)**	**Error**
2	143	721,564 €	705,822 €	15,743 €
3	2,246	398,994 €	398,994 €	- €
4	2,841	113,322 €	113,322 €	- €
5	1,757	445,404 €	445,404 €	- €
6	4,998	701,924 €	684,715 €	17,209 €
7	4,828	233,711 €	233,711 €	- €
8	646	81,079 €	81,079 €	- €
9	2,635	575,033 €	575,033 €	10,000 €
10	3,873	470,763 €	470,763 €	- €
11	834	381,364 €	381,364 €	- €
12	4,738	208,882 €	208,882 €	- €
13	1,695	590,045 €	590,045 €	- €
14	2,589	422,135 €	422,135 €	- €
15	2,917	171,645 €	171,645 €	- €
16	1,434	474,949 €	474,949 €	- €
17	4,077	86,929 €	86,303 €	1,200 €
18	953	293,675 €	293,675 €	- €
19	232	116,948 €	116,948 €	- €
20	1,097	275,418 €	275,418 €	- €
21	166	313,678 €	313,678 €	- €
22	**Total**	7,077,466 €	7,043,888 €	44,151 €
23	**Sample error rate:=D22/B22**			0.6%
24	**Sample standard deviation of errors:=STDEV(D2:D21)----**			5,370 €

The same procedure was followed for the population of stratum 2.

A preliminary sample of 20 operations of stratum 2 yielded an estimate for the standard deviation of errors of 177,582 €:

Stratum 1 – preliminary estimate of standard deviation of errors	5,370 €
Stratum 2 - preliminary estimate of standard deviation of errors	177,582 €

Therefore, the weighted average of the variances of the errors of these two strata is

$$\sigma_w^2 = \frac{2520}{3{,}847} 5{,}370^2 + \frac{1{,}327}{3{,}847} 177{,}582^2 = 10{,}896{,}828{,}862$$

The sample size is given by

$$n = \left(\frac{3{,}847 \times 1.282 \times \sqrt{10{,}896{,}828{,}862}}{83{,}997{,}640 - 16{,}799{,}528} \right)^2 \approx 59$$

The total sample size is given by these 59 operations plus the 5 operation of the 100% sampling stratum, that is, 64 operations.

The allocation of the sample by stratum is as follows:

$$n_1 = \frac{N_1}{N_1 + N_2} \times n = \frac{2{,}520}{3{,}847} \times 59 \approx 39,$$

$$n_2 = n - n_1 = 20$$

and

$$n_3 = N_3 = 5$$

Auditing 39 operations in stratum 1, 20 operations in stratum 2 and 5 operations in stratum 3 will provide the auditor with a total error for the sampled operations. The previous preliminary samples of 20 in stratum 1 and 2 are used as part of the main sample. Therefore, the auditor has only to randomly select 19 further operations in stratum 1. The following table shows the sample results for the 64 operations audited:

Sample results – stratum 1		
A	Sample book value	11,966,658 €
B	Sample total error	190,866 €
C	Sample average error (C=B/39)	4,894 €
D	Sample standard deviation of errors	4,329 €
Sample results – stratum 2		
E	Sample book value	572,607,646 €
F	Sample total error	400,825 €
G	Sample average error (G=F/20)	20,041 €
H	Sample standard deviation of errors	177,582 €
Sample results – stratum 3		
I	Sample book value	584,359,223 €
J	Sample total error	7,240,855 €

K	Sample average error (K=J/5)	1,448,171 €

The following figure illustrates the results for stratum 1:

	A	B	C	D	E	F
1	**Operation ID (1)**	**Book Value (BV) (2)**	**Audited Value (AV) (3)**	**Error (4)**	**Error rate (4)/(2)**	**q (4)-SUM(4)/SUM(2)*(2)**
2	143	721,564 €	705,822 €	15,743 €	2.2%	4,224 €
3	2,246	398,994 €	398,994 €	- €	0.0%	- 6,369 €
4	2,841	113,322 €	113,322 €	- €	0.0%	- 1,809 €
5	1,757	445,404 €	445,404 €	- €	0.0%	- 7,110 €
6	4,998	701,924 €	684,715 €	17,209 €	2.5%	6,004 €
7	4,828	233,711 €	233,711 €	- €	0.0%	- 3,731 €
8	646	81,079 €	81,079 €	- €	0.0%	- 1,294 €
9	2,635	575,033 €	575,033 €	10,000 €	1.7%	821 €
10	3,873	470,763 €	470,763 €	- €	0.0%	- 7,515 €
11	834	381,364 €	381,364 €	- €	0.0%	- 6,088 €
12	4,738	208,882 €	208,882 €	- €	0.0%	- 3,334 €
13	1,695	590,045 €	590,045 €	- €	0.0%	- 9,419 €
14	2,589	422,135 €	422,135 €	- €	0.0%	- 6,739 €
15	2,917	171,645 €	171,645 €	- €	0.0%	- 2,740 €
16	1,434	474,949 €	474,949 €	- €	0.0%	- 7,582 €
17	4,077	86,929 €	86,303 €	1,200 €	1.4%	- 188 €
18	953	293,675 €	293,675 €	- €	0.0%	- 4,688 €
19	232	116,948 €	116,948 €	- €	0.0%	- 1,867 €
20	1,097	275,418 €	275,418 €	- €	0.0%	- 4,397 €
21	166	313,678 €	313,678 €	- €	0.0%	- 5,007 €
22	(...)	(...)	(...)	(...)	(...)	(...)
39	1699	393,458 €	392,487 €	972 €	0.2%	- 5,309 €
40	2392	294,702 €	290,764 €	3,938 €	1.3%	- 767 €
41	**Total**	11,956,658 €	11,765,792 €	190,866 €		
42	**Sample standard deviation:=STDEV()---->**			4,329 €		76,286 €
43						

In the mean-per-unit estimation, extrapolating the error for the two sampling strata is done by multiplying the sample average error by the population size. The sum of these two figures has to be added to the error found in the 100% sampling strata, in order to project error to the population:

$$EE_1 = \sum_{h=1}^{3} N_h \times \frac{\sum_{i=1}^{n_h} E_i}{n_h} = 2{,}520 \times 4{,}894 + 1{,}327 \times 20{,}041 + 7{,}240{,}855$$

$$= 46{,}168{,}474$$

An alternative estimated result using ratio estimation is obtained by multiplying the average error rate observed in the stratum sample by the book value at the stratum level (for the two sampling strata). Then, the sum of these two figures has to be added to the error found in the 100% sampling strata, in order to project error to the population:

$$EE_2 = \sum_{h=1}^{3} BV_h \times \frac{\sum_{i=1}^{n_h} E_i}{\sum_{i=1}^{n_h} BV_i}$$
$$= 2,168,367,291 \times \frac{190,866}{11,966,958} + 1,447,155,510 \times \frac{400,825}{572,607,646}$$
$$+ 7,240,855 = 42,838,924.$$

The projected error rate is computed as the ratio between the projected error and the book value of the population (total expenditure). Using the mean-per-unit estimation the projected error rate is

$$r_1 = \frac{46,168,142}{4,199,882,024} = 1.10\%$$

and using the ratio estimation is:

$$r_2 = \frac{42,838,056}{4,199,882,024} = 1.02\%$$

In both cases, the projected error is smaller than the materiality level. However, final conclusions can only be drawn after taking into account the sampling error (precision). Notice, that the only sources of sampling error are strata 1 and 2, since the high-value stratum is submitted to a 100% sampling. In what follows, only the two sampling strata are considered.

Given the standard deviations of errors in the sample of both strata (table with sample results), the weighted average of the variance of errors for the whole set of strata is:

$$s_w^2 = \sum_{i=1}^{2} \frac{N_h}{N} s_{eh}^2 = \frac{2520}{3,847} \times 4,329^2 + \frac{1,327}{3,847} \times 177,582^2 = 10,890,214,986.$$

Therefore, the precision of the absolute error is given by the following formula:

$$SE_1 = N \times z \times \frac{s_w}{\sqrt{n}} = 3,847 \times 1.282 \times \frac{\sqrt{10,890,214,986}}{\sqrt{59}} = 67,004,263.$$

For the ration estimation, it is necessary to create the variable

$$q_{ih} = E_{ih} - \frac{\sum_{i=1}^{n_h} E_{ih}}{\sum_{i=1}^{n_h} BV_{ih}} \times BV_{ih}.$$

The illustration for stratum 1 is in the last column of the previous table (column F). For instance the value in cell F2 is given by the value of the error of the first operation (15,743 €) minus the sum of sample errors, in column E, 190,866 € (“**:=SUM(D2:D40)**”) divided by the sum of sample book values, in column B, 11,956,658 € (“**:=SUM(B2:B40)**”), multiplied by the book value of the operation (721,564 €):

$$q_{11} = 0 - \frac{190{,}866}{11{,}956{,}658} \times 721{,}564 = 4{,}224.$$

The standard deviation of this variable for stratum 1 is $s_{q1} = 76{,}286$ (computed in MS Excel as **“:=STDEV(F2:F40)**”). Using the methodology just described, the standard deviation for stratum 2 is $s_{q2} = 92{,}907$. Therefore the weighted sum of the variances of q_{ih}:

$$s_{qw}^2 = \sum_{h=1}^{3} \frac{N_h}{N} s_{qh}^2 = \frac{2{,}520}{3{,}847} \times 76{,}286^{\ 2} + \frac{1{,}327}{3{,}847} \times 92{,}907^2 = 6{,}789{,}595{,}201.$$

The precision for ratio estimation is given by

$$SE_2 = N \times z \times \frac{s_{qw}}{\sqrt{n}} = 3{,}847 \times 1.282 \times \frac{\sqrt{6{,}789{,}595{,}201}}{\sqrt{59}} = 52{,}906{,}131.$$

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

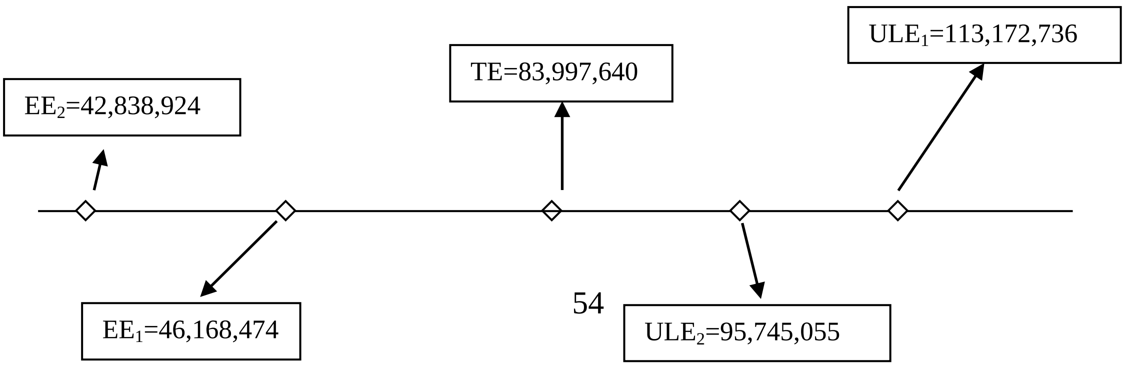
Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

$$ULE_1 = EE_1 + SE_1 = 46{,}168{,}474 + 67{,}004{,}263 = 113{,}172{,}736$$

or

$$ULE_2 = EE_2 + SE_2 = 42{,}838{,}924 + 52{,}906{,}131 = 95{,}745{,}055$$

Finally, comparing to the materiality threshold of 2% of the total book value of the population (2% x 4,199,882,024 € = 83,997,640 €) with the projected results we observe that the maximum tolerable error is larger than the projected errors (using both methods), but smaller than the upper limit. Therefore, additional work (as described in Section 5.11) is needed as there is not enough evidence to support that the population is not materially misstated.



7.1.3 *Simple random sampling – two periods*

7.1.3.1 Introduction

The audit authority may decide to carry out the sampling process in several periods during the year (typically two semesters). The major advantage of this approach is not related with sample size reduction, but mainly allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

With this approach the year population is divided in two sub-populations, each one corresponding to the operations and expenditure of each semester. Independent samples are drawn for each semester, using the standard simple random sampling approach.

7.1.3.2 Sample size

First semester

At the first period of auditing (e.g. semester) the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors for in each semester:

$$\sigma_w^2 = \frac{N_1}{N}\sigma_{e1}^2 + \frac{N_2}{N}\sigma_{e2}^2$$

and σ_{et}^2 is the variance of errors in each period t (semester). The variance of the errors for each semester is computed as an independent population as

$$\sigma_{et}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(E_{ti} - \bar{E}_t)^2\,, t = 1{,}2$$

where E_{ti} represent the individual errors for units in the sample of semester *t* and $\bar{E}_t$ represent the mean error of the sample in semester *t*.

Note that the values for the expected variances in both semesters values have to be set using professional judgments and must be based on historical knowledge. The option to implement a preliminary/pilot sample of low sample size as previously presented for the standard simple random sampling method is still available, but can only be performed for the first semester. In fact, at the first moment of observation expenditure for the second semester has not yet taken place and no objective data (besides historical) is available. If pilot samples are implemented, they can, as usual, subsequently be used as a part of the sample chosen for audit.

If no historical data or knowledge is available to assess the variability of data in the second semester, a simplified approach can be used, computing the global sample size as

$$n = \left(\frac{N \times z \times \sigma_{e1}}{TE - AE}\right)^2$$

Note that in this simplified approach only information about the variability of errors in the first period of observation is needed. The underlying assumption is that the variability of errors will be of similar magnitude in both semesters.

Also note that the formulas for sample size calculation require values for N_1 and N_2, i.e. number of operation in the population of the first and second semesters. When calculating sample size, the value for N_1 will be known, but the value of N_2 will be unknown and has to be imputed according to the expectations of the auditor (also based on historical information). Usually, this does not constitute a problem as all the operations active in the second semester already exist in the first semester and therefore $N_1 = N_2$.

Once the total sample size, n, is computed the allocation of the sample by semester is as follows:

$$n_1 = \frac{N_1}{N} n$$

and

$$n_2 = \frac{N_2}{N} n$$

Second semester

At the first observation period some assumptions were made relatively the following observation periods (typically the next semester). If characteristics of the population in the following periods differ significantly from the assumptions, sample size for the following period may have to be adjusted.

In fact, at the second period of auditing (e.g. semester) more information will be available:

- The number of operations active in the semester N_2 is correctly known;
- The sample variance of errors s_{e1} calculated from the sample of the first semester is already available;

- The standard deviation of errors for the second semester σ_{e2} can now be more accurately assessed using real data.

If these parameters are not dramatically different from the ones estimated at the first semester using the expectations of the analyst, the originally planned sample size, for the second semester (n_2), won't require any adjustments. Nevertheless if the auditor finds that initial expectations significantly differ from the real population characteristics, the sample size may have to be adjusted in order to account for these inaccurate estimates. In this case, the sample size of the second semester should be recalculated using

$$n_2 = \frac{\left(z. N_2\ .\sigma_{e2}\right)^2}{(TE - AE)^2 - z^2 . \frac{N_1^2}{n_1} . s_{e1}^2}$$

where s_{e1} is the standard-deviation of errors calculated from the sample of the first semester and σ_{e2} an estimate of the standard-deviation of errors in the second semester based on historical knowledge (eventually adjusted by information from the first semester) or a preliminary/pilot sample of the second semester.

7.1.3.3 Projected error

Based on the two sub-samples of each semester, the projected error at the level of the population can be computed through the two usual methods: mean-per-unit estimation and ratio estimation.

Mean-per-unit estimation
In each semester multiply the average error per operation observed in the sample by the number of operations in the population (N_t); then sum the results obtained for both semesters, yielding the projected error:

$$EE_1 = \frac{N_1}{n_1} \sum_{i=1}^{n_1} E_{1i} + \frac{N_2}{n_2} \sum_{i=1}^{n_2} E_{2i}$$

Ratio estimation
In each semester multiply the average error rate observed in the sample by the population book value of the respective semester (BV_t):

$$EE_2 = BV_1 \times \frac{\sum_{i=1}^{n_1} E_{1i}}{\sum_{i=1}^{n_1} BV_{1i}} + BV_2 \times \frac{\sum_{i=1}^{n_2} E_{2i}}{\sum_{i=1}^{n_2} BV_{2i}}$$

The sample error rate in each semester is just the division of the total amount of error in the sample of the semester by the total amount of expenditure in the same sample.

The choice between the two methods should be based upon the considerations presented for the standard simple random sampling method.

7.1.3.4 Precision

As for the standard method, precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). It is calculated differently according to the method that has been used for extrapolation.

Mean-per-unit estimation (absolute errors)
The precision is given by the following formula

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2}\right)}$$

where s_{et} is the standard-deviation of errors in the sample of semester t, (now calculated from the same samples used to project the errors to the population)

$$s_{et}^2 = \frac{1}{n_t - 1} \sum_{i=1}^{n_t} (E_{ti} - \bar{E}_t)^2$$

Ratio estimation (error rates)
The precision is given by the following formula

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{q1}^2}{n_1} + N_2^2 \times \frac{s_{q2}^2}{n_2}\right)}$$

where s_{qt} is the standard deviation of the variable q in the sample of semester t, where

$$q_{ti} = E_{ti} - \frac{\sum_{i=1}^{n_t} E_{ti}}{\sum_{i=1}^{n_t} BV_{ti}} \times BV_{ti}.$$

7.1.3.5 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 7.1.1.5.

7.1.3.6 Example

An AA decided to spread the audit workload in two periods. At the end of the first semester AA considers the population divided into two groups corresponding to both semesters. At the end of the first semester, the characteristics of the population are the following:

Declared expenditure at the end of first semester	1,237,952,015 €
Size of population (operations - first semester)	3,852

Based on the experience, the AA knows that usually all the operations included in the programmes at the end of the reference period are already active in the population of the first semester. Furthermore, it is expected that the declared expenditure at the end of the first semester represents about 30% of the total declared expenditure at the end of the reference year. Based on these assumptions a summary of the population is described in the following table:

Declared expenditure at the end of first semester	1,237,952,015 €
Declared expenditure at the end of the second semester (predicted)	2,888,554,703 €
Size of population (operations - period 1)	3,852
Size of population (operations – period 2, predicted)	3,852

The system audits carried out by the audit authority have yielded a moderate assurance level. Therefore, sampling this programme can be done with a confidence level of 80%.

At the first period, the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors in each semester:

$$\sigma_w^2 = \frac{N_1}{N}\sigma_{e1}^2 + \frac{N_2}{N}\sigma_{e2}^2$$

and σ_{et}^2 is the variance of errors in each period t (semester). The variance of the errors for each semester is computed as an independent population as

$$\sigma_{et}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(E_{ti} - \bar{E}_t)^2 \, , t = 1{,}2$$

where E_{ti} represent the individual errors for units in the sample of semester t and $\bar{E}_t$ represent the mean error of the sample in semester t.

Since the value of σ_{et}^2 is unknown, the AA decided to draw a preliminary sample of 20 operations at the end of first semester of the current year. The sample standard deviation of errors in this preliminary sample at first semester is 72,091 €. Based on professional judgement and knowing that usually the expenditure in second semester is larger than in first semester, the AA has made a preliminary prediction of standard deviation of errors for the second semester to be 40% larger than in first semester, that is, 100,475 €. Therefore, the weighted average of the variances of the errors is:

$$\begin{aligned}\sigma_w^2 &= \frac{N_1}{N_1 + N_2}\sigma_{e1}^2 + \frac{N_2}{N_1 + N_2}\sigma_{e2}^2 \\ &= \frac{3852}{3852 + 3852} \times 72{,}091^2 + \frac{3852}{3852 + 3852} \times 100{,}475^2 \\ &= 7{,}646{,}168{,}953.\end{aligned}$$

Note that the population size in each semester is equal to the number of active operations (with expenditure) in each semester.

At the first semester the global sample size planned for the whole year is:

$$n = \left(\frac{(N_1 + N_2) \times z \times \sigma_w}{TE \ - AE}\right)^2$$

where z is 0.842 (coefficient corresponding to a 60% confidence level), TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value. The total book value comprises the true book value at the end of the first semester plus the predicted book value for the second semester (1,237,952,015 € + 2,888,554,703 € = 4,126,506,718 €), which means that tolerable error is 2% x 4,126,506,718 € = 82,530,134 €. The preliminary sample on the first semester population yields a sample error rate of 0.6%. The audit authority expects this error rate to remain constant all over the year. Thus AE, the anticipated error, is 0.6% x 4,126,506,718 € = 24,759,040 €. The planned sample size for the whole year is:

$$n = \left(\frac{(3852 + 3852) \times 0.842 \times \sqrt{7{,}646{,}168{,}953}}{82{,}530{,}134 - 24{,}759{,}040} \right)^2 \approx 97$$

The allocation of the sample by semester is as follows:

$$n_1 = 0.5\, n \approx 49$$

and

$$n_2 = n - n_1 = 49$$

The first semester sample yielded the following results:

Sample book value - first semester	13,039,581 €
Sample total error - first semester	199,185 €
Sample standard deviation of errors - first semester	69,815 €

At the end of the second semester more information is available, in particular, the number of operations active in the second semester is correctly known, the sample variance of errors s_{e1} calculated from the sample of the first semester is already available and the standard deviation of errors for the second semester σ_{e2} can now be more accurately assessed using a preliminary sample of real data.

The AA realises that the assumption made at the end of the first semester on the total number of operations remains correct. Nevertheless, there are two parameters for which updated figures should be used.

Firstly, the estimate of the standard deviation of errors based on the first semester sample of 49 operations yielded an estimate of 69,815 €. This new value should now be used to reassess the planned sample size. Secondly, based on a new preliminary sample of 20 operations of the second semester population, the audit authority estimates the standard deviation of errors for the second semester to be 108,369 € (close to the predicted value at the end of the first period, but more accurate). We conclude that the standard deviations of errors of both semesters, used to plan the sample size, are close to the values obtained at the end of the first semester. Nevertheless, the audit authority has chosen to recalculate the sample size using the available updated data. As a result, the sample for the second semester is revised.

Further, the predicted total book value of the second semester population should be replaced by the real one, 2,961,930,008 €, instead of the predicted value of 2,888,554,703 €.

Parameter	End of first semester	End of second semester
Standard deviation of errors in the first semester	72,091 €	69,815 €
Standard deviation of errors in the second semester	100,475 €	108,369 €
Total expenditure in the second semester	2,888,554,703 €	2,961,930,008 €

Taking into account these adjustments, the recalculated sample size of the second semester is

$$n_2 = \frac{\left(z \times N_2 \times \sigma_{e2}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s_{e1}^2}$$
$$= \frac{(0.842 \times 3{,}852 \times 108{,}369)^2}{(83{,}997{,}640 - 25{,}199{,}292)^2 - 0.842^2 \times \frac{3{,}852^2}{49} \times 69{,}815^2} = 52$$

Auditing 49 operations is the first semester plus these 52 operations in second semester will provide the auditor with information on the total error for the sampled operations. The previous preliminary sample of 20 operations is used as part of the main sample. Therefore, the auditor has only to select 32 further operations in second semester.

The second semester sample yielded the following results:

Sample book value - second semester	34,323,574 €
Sample total error - second semester	374,790 €
Sample standard deviation of errors - second semester	59,489 €

Based on both samples, the projected error at the level of the population can be computed through the two usual methods: mean-per-unit estimation and ratio estimation. The first method comprises multiplying the average error per operation observed in the sample by the number of operations in the population (N_t); then sum the results obtained for both semesters, yielding the projected error:

$$EE_1 = \frac{N_1}{n_1}\sum_{i=1}^{49} E_{1i} + \frac{N_2}{n_2}\sum_{i=1}^{52} E_{2i} = \frac{3{,}852}{49} \times 199{,}185 + \frac{3{,}852}{52} \times 374{,}790$$
$$= 43{,}318{,}770$$

The second method comprises multiplying the average error rate observed in the sample by the population book value of the respective semester (BV_t):

$$
\begin{aligned}
EE_2 = BV_1 \times \frac{\sum_{i=1}^{n_1} E_{1i}}{\sum_{i=1}^{n_1} BV_{1i}} + BV_2 \times \frac{\sum_{i=1}^{n_2} E_{2i}}{\sum_{i=1}^{n_2} BV_{2i}} \\
= 1{,}237{,}952{,}015 \times \frac{199{,}185}{13{,}039{,}581} + 2{,}961{,}930{,}008 \times \frac{374{,}790}{34{,}323{,}574} \\
= 51{,}252{,}484
\end{aligned}
$$

Using the mean-per-unit estimation the projected error rate is:

$$
r_1 = \frac{43{,}421{,}670}{1{,}237{,}952{,}015 + 2{,}961{,}930{,}008} = 1.03\%
$$

and using the ratio estimation is:

$$
r_2 = \frac{51{,}252{,}451}{1{,}237{,}952{,}015 + 2{,}961{,}930{,}008} = 1.22\%.
$$

The precision is calculated differently according to the method that has been used for projection. For mean-per-unit estimation, the precision is given by the following formula

$$
\begin{aligned}
SE_1 = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2}\right)} \\
= 0.842 \times \sqrt{3{,}852^2 \times \frac{69{,}815^2}{49} + 3{,}852^2 \times \frac{59{,}489^2}{52}} = 41{,}980{,}051
\end{aligned}
$$

For the ratio estimation, the standard deviation of the variable q has to be calculated (Section 7.1.2.6):

$$
q_{ti} = E_{ti} - \frac{\sum_{i=1}^{n_t} E_{ti}}{\sum_{i=1}^{n_t} BV_{ti}} \times BV_{ti}.
$$

This standard deviation for each semester is, 54,897 € and 57,659 €, respectively. Thus the precision is given by

$$
\begin{aligned}
SE_2 = z \times \sqrt{\left(N_1^2 \times \frac{s_{q1}^2}{n_1} + N_2^2 \times \frac{s_{q2}^2}{n_2}\right)} \\
= 0.842 \times \sqrt{3{,}852^2 \times \frac{54{,}897^2}{49} + 3{,}852^2 \times \frac{57{,}659^2}{52}} = 36{,}325{,}544
\end{aligned}
$$

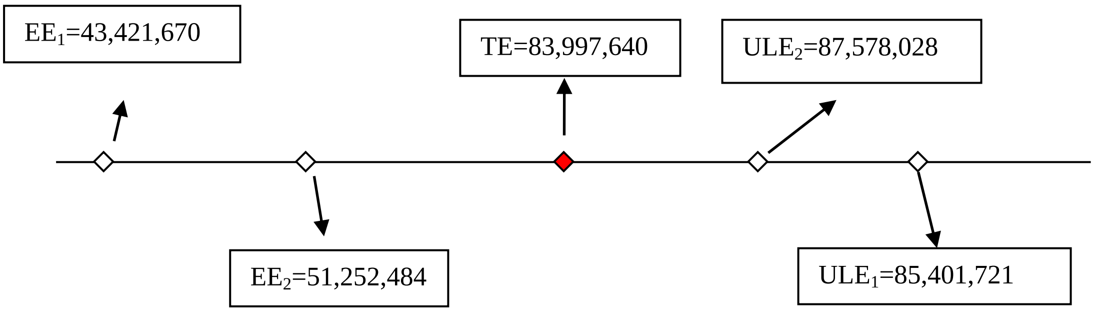
Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

$$ULE_1 = EE_1 + SE_1 = 43{,}421{,}670 + 41{,}980{,}051 = 85{,}401{,}721$$

or

$$ULE_2 = EE_2 + SE_2 = 51{,}252{,}484 + 36{,}325{,}544 = 87{,}578{,}028$$

Finally, comparing to the materiality threshold of 2% of the total book value of the population (2% x 4,199,882,023 € = 83,997,640 €) with the projected results we observe that the maximum tolerable error is larger than the projected errors (using both methods), but smaller than the upper limit. Therefore, additional work (as described in Section 5.11) is needed as there is not enough evidence to support that the population is not materially misstated.



7.2 Difference estimation

7.2.1 Standard approach

7.2.1.1 Introduction

Difference estimation is also a statistical sampling method based on equal probability selection. The method relies on extrapolating the error in the sample and subtracting the projected error to the total declared expenditure in the population in order to assess the correct expenditure in the population (i.e. the expenditure that would be obtained if all the operations in the population were audited).

This method is very close to simple random sampling, having as main difference the use of a more sophisticated extrapolation device.

This method is particularly useful if one wants to project the correct expenditure in the population, if the level of error is relatively constant in the population, and if the book value of different operations tends to be similar (low variability). It tends to be better than MUS when errors have low variability or are weakly or negatively associated with book values. On the other hand, tends to be worse than MUS is errors have strong variability and are positively associated with book values

As all other methods, this method can be combined with stratification (favourable conditions for stratification are discussed in Section 6.2 and specific formulas are presented in section 7.2.2).

7.2.1.2 Sample size

Computing sample size n within the framework of difference estimation relies on exactly the same information and formulas used in simple random sampling:

- Population size N
- Confidence level determined from systems audit and the related coefficient z from a normal distribution (see Section 6.4)
- Maximum tolerable error TE (usually 2% of the total expenditure)
- Anticipated error AE chosen by the auditor according to professional judgment and previous information
- The standard deviation σ_e of the errors.

The sample size is computed as follows:

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

where σ_e is the standard-deviation of errors in the population. Please note that, as discussed in the framework of simple random sampling, this standard-deviation is almost never know in advance and Member States will have to rely either on historical data (standard-deviation of the errors for the population in the past period) or on a preliminary/pilot sample of low sample size (sample size is recommended to be not smaller than 20 to 30 units). Also, note that the pilot sample can subsequently be used as a part of the sample chosen for audit. For additional information on how to calculate this standard-deviation see Section 7.1.1.2.

7.2.1.3 Extrapolation

Based on a randomly selected sample of operations, the size of which has been computed according to the above formula, the projected error at the level of the population can be computed by multiplying the average error observed per operation in the sample by the number of operations in the population, yielding the projected error

$$EE = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$

where E_i represent the individual errors for units in the sample and $\bar{E}$ represent the mean error of the sample.

In a second step the correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected subtracting the projected error (EE) from the book value (BV) in the population (declared expenditure). The projection for the correct book value (CBV) is

$$CBV = BV - EE$$

7.2.1.4 Precision

The precision of the projection (measure of the uncertainty associated with the projection) is given by

$$SE = N \times z \times \frac{s_e}{\sqrt{n}}$$

where s_e is the standard-deviation of errors in the sample (now calculated from the same sample used to project the errors to the population)

$$s_e^2 = \frac{1}{n-1} \sum_{i=1}^{n} (E_i - \bar{E})^2$$

7.2.1.5 Evaluation

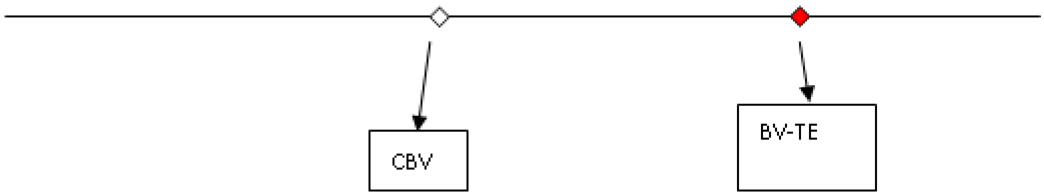
To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE$$

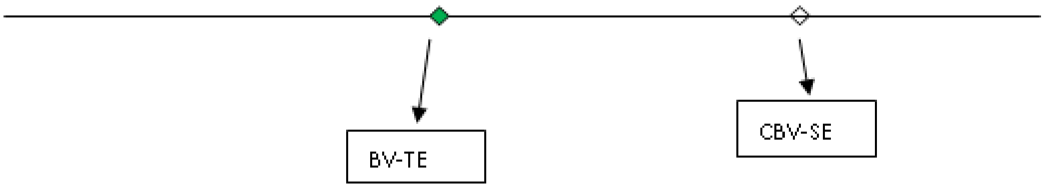
The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE), which corresponds to the materiality level times the book value:

$$BV - TE = BV - 2\% \times BV = 98\% \times BV$$

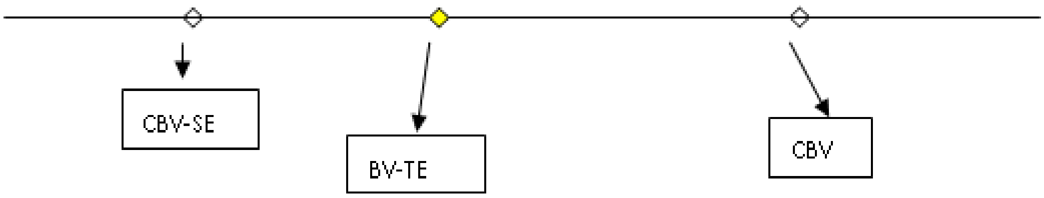
- If $BV - TE$ is larger than CBV the auditor should conclude that there is enough evidence that errors in the programme are larger than materiality threshold:



- If $BV - TE$ is lower than the lower limit $CBV - SE$ than it means there is enough evidence that errors in the programme are lower than materiality threshold.



- If $BV - TE$ is between the lower limit $CBV - SE$ and CBV than it means that additional work is needed as there is not enough evidence to support that the population is not materially misstated. The nature of the additional work needed is discussed in Section 5.11:



7.2.1.6 Example

Let's assume a population of expenditure certified to the Commission in a given year for operations in a programme. The system audits carried out by the audit authority have yielded a high assurance level. Therefore, sampling this programme can be done with a confidence level of 60%.

The following table summarises the population details:

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference year)	4,199,882,024 €

Based on last year's audit the AA expects an error rate of 0.7% (the last year error rate) and estimates a standard deviation of errors of 168,397 €.

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

where z is 0.842 (coefficient corresponding to a 60% confidence level), σ_e is 168,397 €, TE, the tolerable error, is 2% of the book value (maximum materiality level set by the Regulation), i.e. 2% x 4,199,882,024 € = 83,997,640 € and AE, the anticipated error is 0.7%, i.e., 0.7% x 4,199,882,024 € = 29,399,174 €:

$$n = \left(\frac{3{,}852 \times 0.842 \times 168{,}397}{83{,}997{,}640 - 29{,}399{,}174}\right)^2 \approx 101$$

The minimum sample size is therefore 101 operations.

Auditing these 101 operations will provide the auditor with a total error for the sampled operations.

The sample results are summarised in the following table:

Sample book value ("**:=SUM(B2:B102)**")	124,944,535 €
Sample total error ("**:=SUM(D2:D102)**")	1,339,765 €
Sample standard deviation of errors ("**:=STDEV(D2:D102)**")	162,976 €

The following figure illustrates the results of the sampling procedure:

	A	B	C	D
1	**Operation ID (1)**	**Book Value (BV) (2)**	**Audited Value (AV) (3)**	**Error (4)**
2	4371	265,032 €	259,894 €	5,138 €
3	2238	566,474 €	566,474 €	- €
4	4972	388,768 €	388,768 €	- €
5	3050	231,114 €	231,114 €	- €
6	459	869,080 €	869,080 €	- €
7	4232	11,856,372 €	11,856,372 €	- €
8	1371	640,087 €	640,087 €	- €
9	1278	372,259 €	372,259 €	- €
10	4217	626,949 €	626,949 €	- €
11	2381	221,181 €	221,181 €	- €
12	4214	661,927 €	661,927 €	- €
13	2868	611,219 €	611,219 €	- €
14	2350	326,179 €	326,179 €	- €
15	3121	835,153 €	835,153 €	- €
16	2936	114,856 €	114,856 €	- €
17	203	16,098,649 €	16,098,649 €	- €
18	2073	859,992 €	859,992 €	- €
19	1057	272,282 €	272,282 €	- €
20	3270	759,543 €	698,060 €	61,483 €
21	3915	767,864 €	767,864 €	- €
22	**(…)**	**(…)**	**(…)**	**(…)**
101	1482	601,270 €	593,783 €	7,486 €
102	1109	644,971 €	644,971 €	- €
103	**Total:=SUM([Row 2]:[Row 102])**	124,944,535 €	123,604,770 €	1,339,765
104	**Sample standard deviation:=STDEV(D2:D102)----->**			162,976
105				

The projected error at the level of the population is:

$$EE = N \times \frac{\sum_{i=1}^{101} E_i}{n} = 3{,}852 \times \frac{1{,}339{,}765}{101} = 51{,}096{,}780,$$

corresponding to a projected error rate of:

$$r = \frac{51{,}096{,}780}{4{,}199{,}882{,}024} = 1.22\%$$

The correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected subtracting the projected error (EE) from the book value (BV) in the population (declared expenditure). The projection for the correct book value (CBV) is

$$CBV = 4{,}199{,}882{,}024 - 51{,}096{,}780 = 4{,}148{,}785{,}244$$

The precision of the projection is given by

$$SE = N \times z \times \frac{s_e}{\sqrt{n}} = 3{,}852 \times 0.842 \times \frac{162{,}976}{\sqrt{101}} = 52{,}597{,}044.$$

Combining the projected error and the precision it is possible to compute an upper limit for the error rate. This upper limit is the ratio of the upper limit of error to the book value of the population. Therefore, the upper limit for the error rate is:

$$r_{UL} = \frac{EE + SE}{BV} = \frac{51{,}096{,}780 + 52{,}597{,}044}{4{,}199{,}882{,}024} = 1.47\%$$

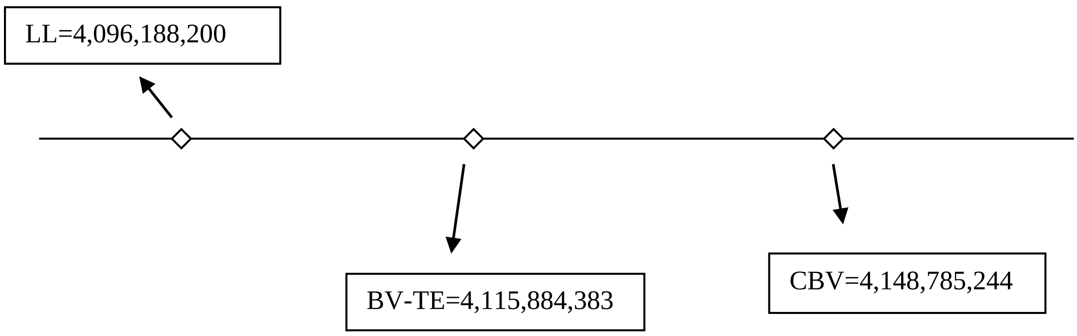
To conclude about the materiality of the errors the lower limit for the correct book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE = 4{,}148{,}785{,}244 - 52{,}597{,}044 = 4{,}096{,}188{,}200$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (*TE*):

$$BV - TE = 4{,}199{,}882{,}024 - 83{,}997{,}640 = 4{,}115{,}884{,}383$$

As $BV - TE$ is between the lower limit $LL = CBV - SE$ and CBV, than additional work is needed in order to prove that the population is not materially misstated. The nature of the additional work needed is discussed in Section 5.11.



7.2.2 Stratified difference estimation

7.2.2.1 Introduction

In stratified difference estimation, the population is divided in sub-populations called strata and independent samples are drawn from each stratum, using the difference estimation method.

The rationale behind stratification and the candidate criteria to implement stratification are the same as presented for simple random sampling (see Section 7.1.2.1). As for simple random sampling, the stratification by level of expenditure per operation is usually a good approach whenever it is expected that the level of error is associated with the level of expenditure.

If stratification by level of expenditure is implemented, and if it is possible to find a few extremely high value operations it is recommended that they are included in a high-value stratum, that will be a 100% audited. In this case, the items belonging to the 100% stratum should be treated separately and the sampling steps will apply only to the population of the low-value items. The reader should be aware that the planned precision for sample size determination should be however based on the total book value of the population. Indeed, as the source of error is the low-value items stratum, but the planned precision is due at population level, the tolerable error and the anticipated error should be calculated at population level, as well.

7.2.2.2 Sample size

The sample size is computed using the same approach as for simple random sampling

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors for the whole set of strata (see Section 7.1.2.2 for further details).

As usual, the variances can be based on historical knowledge or on a preliminary/pilot sample of small sample size. In this later case, the pilot sample can, as usual, subsequently be used as a part of the main sample for audit.

Once the total sample size, n, is computed the allocation of the sample by stratum is as follows:

$$n_h = \frac{N_h}{N} \times n.$$

This is the same general allocation method, also used in simple random sampling, known as proportional allocation. Again, other allocation methods are available and can be applied.

7.2.2.3 Extrapolation

Based on H randomly selected samples of operations, the size of each one has been computed according to the above formula, the projected error at the level of the population can be computed in as:

$$EE = \sum_{h=1}^{H} N_h \frac{\sum_{i=1}^{n_h} E_i}{n_h}.$$

In practice, in each group of the population (stratum) multiply the average of observed errors in the sample by the number of operations in the stratum (N_h) and sum all the results obtained for each stratum.

In a second step the correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected using the following formula:

$$CBV = BV - \sum_{h=1}^{H} N_h \frac{\sum_{i=1}^{n_h} E_i}{n_h}$$

In the above formula: 1) in each stratum calculate the average of observed errors in the sample; 2) in each stratum multiply the average sample error by the stratum size (N_h); 3) sum these results for all the strata; 4) subtract this value from the total book value of the population (BV). The result of the sum is a projection for the correct book value (CBV) in the population.

7.2.2.4 Precision

Remember that precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). In stratified difference estimation is given by the following formula

$$SE = N \times z \times \frac{s_w}{\sqrt{n}}$$

where s_w^2 is the weighted mean of the variance of errors for the whole set of strata calculated from the same sample used to project the errors to the population:

$$s_w^2 = \sum_{i=1}^{H} \frac{N_h}{N} s_{eh}^2, h = 1,2, \dots, H;$$

and s_{eh}^2 is the estimated variance of errors for the sample of stratum *h*

$$s_{eh}^2 = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} (E_{hi} - \bar{E}_h)^2 \text{ , } h = 1,2, \dots, H$$

7.2.2.5 Evaluation

To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE)

$$BV - TE = BV - 2\% \times BV = 98\% \times BV$$

Finally, audit conclusions should be drawn using exactly the same approach presented in Section 7.2.1.5 for standard difference estimation.

7.2.2.6 Example

Let us assume a population of expenditure certified to the Commission in a given year for operations in a group of programmes. The management and control system is shared by the group of programmes and the system audits carried out by the audit authority have yielded a high assurance level. Therefore, sampling this programme can be done with a confidence level of 60%.

The AA has reasons to believe that there are substantial risks of error for high value operations, whatever the programme they belong to. Further, there are reasons to expect that there are different error rates across the programmes. Bearing in mind all this information, the AA decides to stratify the population by programme and by expenditure (isolating in a 100% sampling stratum all the operations with book value larger than the materiality).

The following table summarizes the available information:

Population size (number of operations)	4,872
Population size – stratum 1 (number of operations in programme 1)	1,520
Population size – stratum 2 (number of operations in programme 2)	3,347
Population size – stratum 3 (number of operations with BV > materiality level)	5

Book value (sum of the expenditure in the reference year)	6,440,727,190 €
Book value – stratum 1 (total expenditure in programme 1)	3,023,598,442 €
Book value – stratum 2 (total expenditure in programme 2)	2,832,769,525 €
Book value – stratum 3 (total expenditure of operations with BV > Materiality level)	584,359,223 €

The 100% sampling stratum containing the 5 high-value operation should be treated separately as stated in section 7.2.2.1. Therefore, hereafter, the value of N corresponds to the total number of operations in the population, deducted of the number of the operations included in the 100% sampling stratum, i.e. 4,867 (= 4,872 – 5) operations.

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where z is 0.842 (coefficient corresponding to a 60% confidence level) and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value, i.e. 2% x 6,440,727,190 € = 128,814,544 €. Based on previous year experience and on the conclusion of the report on managing and control systems the AA expects an error rate not larger than 0.4%, Thus AE, the anticipated error, is 0.4%, i.e., 0.4% x 6,440,727,190 € = 25,762,909 €.

Since the third stratum is a 100% sampling stratum, the sample size for this stratum is fixed and is equal to the size of the population, that is, the 5 high-value operations. The sample size for the remaining two strata is computed using the above formula, where σ_w^2 is the weighted average of the variances of the errors for the two remaining strata:

$$\sigma_w^2 = \sum_{i=1}^{2} \frac{N_h}{N} \sigma_{eh}^2, h = 1,2;$$

and σ_{eh}^2 is the variance of errors in each stratum. The variance of the errors is computed for each stratum as an independent population as

$$\sigma_{eh}^2 = \frac{1}{n_h^p - 1} \sum_{i=1}^{n_h^p} (E_{hi} - \bar{E}_h)^2 , h = 1,2, \dots, H$$

where E_{hi} represent the individual errors for units in the sample of stratum h and $\bar{E}_h$ represent the mean error of the sample in stratum h. A preliminary sample of 20 operations of stratum 1 yielded an estimate for the standard deviation of errors of 21,312 €:

	A	B	C	D
1	**Operation ID**	**Book Value (BV)**	**Audited Value (AV)**	**Error**
2	143	626,949 €	626,949 €	- €
3	2,246	265,032 €	265,032 €	- €
4	2,841	372,259 €	372,259 €	- €
5	1,757	611,219 €	609,878 €	1,342 €
6	4,998	566,474 €	566,474 €	- €
7	4,828	231,114 €	231,114 €	- €
8	646	326,179 €	326,179 €	- €
9	2,635	11,856,372 €	11,856,372 €	- €
10	3,873	221,181 €	221,181 €	- €
11	834	388,768 €	388,768 €	- €
12	4,738	114,856 €	114,856 €	- €
13	1,695	759,543 €	759,543 €	- €
14	2,589	272,282 €	272,282 €	- €
15	2,917	16,098,649 €	16,098,649 €	- €
16	1,434	661,927 €	661,927 €	- €
17	4,077	767,864 €	672,495 €	95,370 €
18	953	869,080 €	869,080 €	- €
19	232	835,153 €	835,153 €	- €
20	1,097	859,992 €	859,992 €	- €
21	166	640,087 €	640,087 €	- €
22	**Total**	37,344,981 €	37,248,270 €	96,712 €
23	**Sample error rate:=D22/B22**			0.3%
24	**Sample standard deviation of errors:=STDEV(D2:D21)---->**			21,312 €
25				

The same procedure was followed for the population of stratum 2. A preliminary sample of 20 operations of stratum 2 yielded an estimate for the standard deviation of errors of 215,546 €:

Stratum 1 – preliminary estimate of standard deviation of errors	21,312 €
Stratum 2 - preliminary estimate of standard deviation of errors	215,546 €

Therefore, the weighted mean of the variances of the errors of these two strata is

$$\sigma_w^2 = \frac{1{,}520}{4{,}867} \times 21{,}312^2 + \frac{3{,}347}{4{,}867} 215{,}546^2 = 32{,}059{,}168{,}205$$

The minimum sample size is given by:

$$n = \left(\frac{4{,}867 \times 0.845 \times \sqrt{32{,}092{,}103{,}451}}{128{,}814{,}544 \ - 25{,}762{,}909} \right)^2 \approx 51$$

These 51 operations are allocated by stratum as follows:

$$n_1 = \frac{1{,}520}{4{,}867} \times 51 \approx 16,$$

$$n_2 = n - n_1 = 35$$

and

$$n_3 = N_3 = 5$$

The total sample size is therefore 56 operations:

- 16 operations of stratum 1 preliminary sample, plus
- 35 operations of stratum 2 (the 20 preliminary sample operations plus an additional sample of 15 operations); plus
- 5 high-value operations.

The following table shows the sample results for the whole sample of 60 operations:

Sample results – stratum 1		
A	Sample book value	37,344,981 €
B	Sample total error	77,376 €
C	Sample average error (C=B/16)	4,836 €
D	Sample standard deviation of errors	16,783 €
Sample results – stratum 2		
E	Sample book value	722,269,643 €
F	Sample total error	264,740 €
G	Sample average error (G=F/35)	7,564 €
H	Sample standard deviation of errors	117,335 €
Sample results -100% audit stratum		
I	Sample book value	584,359,223 €
J	Sample total error	7,240,855 €
K	Sample average error (I=J/5)	1,448,171 €

Projecting the error for the two sampling strata is calculated by multiplying the sample average error by the population size. The sum of these two figures, added to the error found in the 100% sampling stratum, is the expected error at population level:

$$EE = \sum_{h=1}^{3} 1520 \times 4{,}836 + 3{,}347 \times 7{,}564 + 7{,}240{,}855 = 39{,}908{,}283$$

The projected error rate is computed as the ratio between the extrapolated error and the book value of the population (total expenditure):

$$r_1 = \frac{39{,}908{,}283}{6{,}440{,}727{,}190} = 0.62\%$$

The correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected using the following formula:

$$CBV = BV - EE = 6{,}440{,}727{,}190 - 39{,}908{,}283 = 6{,}400{,}818{,}907$$

Given the standard deviations of errors in the sample of both strata (table with sample results), the weighted mean of the variance of errors for the whole set of sampling strata is:

$$s_w^2 = \sum_{h=1}^{2} \frac{N_h}{N} s_{eh}^2 = \frac{1{,}520}{4{,}867} \times 16{,}783^2 + \frac{3{,}347}{4{,}867} \times 117{,}335^2 = 9{,}555{,}777{,}062$$

The precision of the projection is given by

$$SE = N \times z \times \frac{s_w}{\sqrt{n}} = 4{,}867 \times 0.842 \times \frac{\sqrt{9{,}555{,}777{,}062}}{\sqrt{51}} = 56{,}094{,}639$$

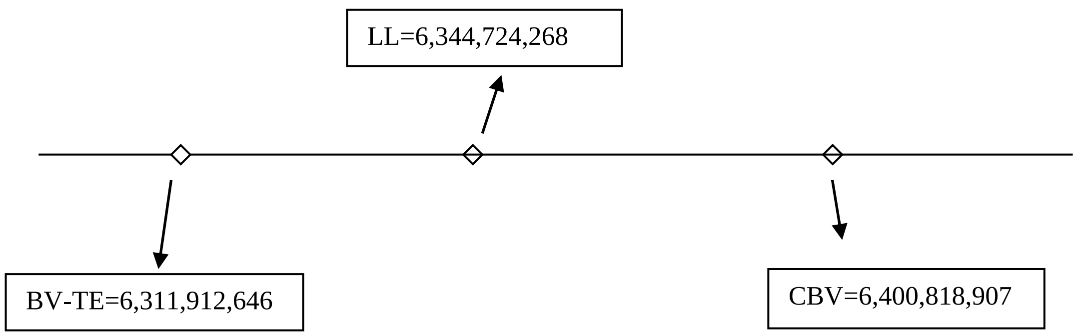
To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE = 6{,}400{,}818{,}907 - 56{,}094{,}639 = 6{,}344{,}724{,}268$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE):

$$BV - TE = 6{,}440{,}727{,}190 - 128{,}814{,}544 = 6{,}311{,}912{,}646$$

Since $BV - TE$ is lower than the lower limit $CBV - SE$ than there is enough evidence that errors in the programme are lower than materiality threshold.



7.2.3 Difference estimation – two periods

7.2.3.1 Introduction

The audit authority may decide to carry out the sampling process in several periods during the year (typically two semesters). The major advantage of this approach is not related with sample size reduction, but mainly allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

With this approach the year population is divided in two sub-populations, each one corresponding to the operations and expenditure of each semester. Independent samples are drawn for each semester, using the standard simple random sampling approach.

7.2.3.2 Sample size

The sample size is computed using the same approach as for simple random sampling in two semesters. See Section 7.1.3.2 for further details.

7.2.3.3 Extrapolation

Based on the two sub-samples of each semester, the projected error at the level of the population can be computed as:

$$EE = N_1.\frac{\sum_{i=1}^{n_1} E_{1i}}{n_1} + N_2.\frac{\sum_{i=1}^{n_2} E_{2i}}{n_2}$$

In practice, in each semester multiply the average of observed errors in the sample by the number of operations in the population (N_t) and sum the results obtained for both semesters.

In a second step the correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected using the following formula:

$$CBV = BV - EE$$

where BV is the yearly book value (including the two semesters) and EE the above projected error.

7.2.3.4 Precision

Remember that precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). It is given by the following formula

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2}\right)}$$

where s_{et} is the standard-deviation of errors in the sample of semester *t*, (now calculated from the same samples used to project the errors to the population)

$$s_{et}^2 = \frac{1}{n_t - 1} \sum_{i=1}^{n_t} (E_{ti} - \bar{E}_t)^2$$

7.2.3.5 Evaluation

To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (*TE*)

$$BV - TE = BV - 2\% \times BV = 98\% \times BV$$

Finally, audit conclusions should be drawn using exactly the same approach presented in Section 7.2.1.5 for standard difference estimation.

7.2.3.6 Example

An AA has decided to split the audit workload between the two semesters of the year. At the end of the first semester the characteristics of the population are the following:

Declared expenditure (DE) at the end of first semester	1,237,952,015 €
Size of population (operations - first semester)	3,852

Based on the past experience, the AA knows that usually all the operations included in the programmes at the end of the reference period are already active in the population of the first semester. Further it is expected that the declared expenditure at the end of the first semester represents about 30% of the total declared expenditure at the end of the reference year. Based on these assumptions a summary of the population is described in the following table:

Declared expenditure (DE) at the end of first semester	1,237,952,015 €
Declared expenditure (DE) at the end of the second semester (predicted)	2,888,554,703 €
Size of population (operations - period 1)	3,852
Size of population (operations – period 2, predicted)	3,852

The system audits carried out by the audit authority have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%.

At the end of the first semester the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors for in each semester:

$$\sigma_w^2 = \frac{N_1}{N}\sigma_{e1}^2 + \frac{N_2}{N}\sigma_{e2}^2$$

and σ_{et}^2 is the variance of errors in each period t (semester). The variance of the errors for each semester is computed as an independent population as

$$\sigma_{et}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(E_{ti} - \bar{E}_t)^2\,, t = 1,2$$

where E_{ti} represent the individual errors for units in the sample of semester t and $\bar{E}_t$ represent the mean error of the sample in semester t.

Since the value of σ_{et}^2 is unknown, the AA decided to draw a preliminary sample of 20 operations at the end of first semester of the current year. The sample standard deviation of errors in this preliminary sample at first semester is 69,534 €. Based on professional judgement and knowing that usually the expenditure in second semester is larger than in first, the AA has made a preliminary prediction of standard deviation of errors for the second semester to be 20% larger than in first semester, that is, 83,441 €. Therefore, the weighted average of the variances of the errors is:

$$\sigma_w^2 = \frac{N_1}{N_1 + N_2}\sigma_{e1}^2 + \frac{N_2}{N_1 + N_2}\sigma_{e2}^2 = 0.5 \times 69{,}534^2 + 0.5 \times 83{,}441^2 = 5{,}898{,}672{,}130.$$

Note that the population size in each semester is equal to the number of active operations (with expenditure) in each semester.

At the end of first semester the global sample size for the whole year is:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted average of the variances of the errors for the whole set of strata (see Section 7.1.2.2 for further details), z is 1.645 (coefficient corresponding to a 90% confidence level), and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value. The total book value comprises the true book value at the end of the first semester plus the predicted book value for the second semester 4,126,506,718, which means that tolerable error is 2% x 4,126,506,718 € = 82,530,134 €. The preliminary sample on the first semester population yields a sample error rate of 0.6%. The audit authority expects these error rate remains constant all over the year. Thus AE, the anticipated error, is 0.6% x 4,126,506,718 € = 24,759,040 €. The sample size for the whole year is:

$$n = \left(\frac{3852 \times 2 \times 1.645 \times \sqrt{5{,}898{,}672{,}130}}{82{,}530{,}134 - 24{,}759{,}040}\right)^2 \approx 284$$

The allocation of the sample by semester is as follows:

$$n_1 = 0.5\, n = 142$$

and

$$n_2 = n - n_1 = 142$$

The first semester sample yielded the following results:

Sample book value - first semester	41,009,806 €
Sample total error - first semester	577,230 €
Sample standard deviation of errors - first semester	65,815 €

At the end of the second semester more information is available, in particular, the number of operations active in the second semester is correctly known, the sample variance of errors s_{e1} calculated from the sample of the first semester is already available and the standard deviation of errors for the second semester σ_{e2} can now be more accurately assessed using a preliminary sample of real data.

The AA realises that the assumption made at the end of the first semester on the total number of operations remains correct. Nevertheless, there are two parameters for which updated figures should be used.

Firstly, the estimate of the standard deviation of errors based on the first semester sample of 142 operations yielded an estimate of 65,815 €. This new value should now be used to reassess the planned sample size. Secondly, based on a new preliminary sample of 20 operations of the second semester population, the audit authority estimates the standard deviation of errors for the second semester to be 107,369 € (faraway of the predicted value at the end of the first period). We conclude that the standard deviation of errors in the first semester used to plan the sample size is close to the value obtained at the end of the first semester. Nevertheless, the standard deviation of error in the second semester used to plan the sample size is far away from the figure given by the new preliminary sample. As a result, the sample for the second semester should be revised.

Further, the predicted total book value of the second semester population should be replaced by the real one, 5,202,775,175 €, instead of the predicted value of 2,888,554,703 €.

Parameter	End of first semester	End of second semester
Standard deviation of errors in the first semester	69,534 €	65,815 €
Standard deviation of errors in the second semester	83,441 €	107,369 €
Total expenditure in the second semester	2,888,554,703 €	5,202,775,175 €

Taking into consideration these two adjustments, the recalculated sample size of the second semester is

$$n_2 = \frac{\left(z \times N_2 \ \times \sigma_{e2}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s_{e1}^2}$$
$$= \frac{(1.645 \ \times 3{,}852 \times 107{,}369)^2}{(128{,}814{,}544 \ - 38{,}644{,}363)^2 - 1.645^2 \times \frac{3{,}852^2}{142} \times 65{,}815^2} \approx 68$$

Auditing the 142 operations in the first semester plus these 68 operations in second semester will provide the auditor with information total error for the sampled operations. The previous preliminary sample of 20 operations is used as part of the main sample. Therefore, the auditor has only to select 48 further operations in second semester.

The second semester sample yielded the following results:

Sample book value - second semester	59,312,212 €
Sample total error - second semester	588,336 €
Sample standard deviation of errors - first semester	53,489 €

Based on both samples, the projected error at the level of the population can be computed as:

$$EE = N_1 \times \frac{\sum_{i=1}^{n_1} E_{1i}}{n_1} + N_2 \times \frac{\sum_{i=1}^{n_2} E_{2i}}{n_2} = 3{,}852 \times \frac{577{,}230}{142} + 3{,}852 \times \frac{588{,}336}{68} = 48{,}985{,}884$$

In a second step the correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected using the following formula:

$$CBV = BV - EE = 6{,}440{,}727{,}190 - 48{,}985{,}884 = 6{,}391{,}741{,}306$$

where BV is the yearly book value (including the two semesters) and EE the above projected error.

The precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation) and it is given by the following formula:

$$SE = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2}\right)}$$

$$= 1.645 \times \sqrt{\left(3852^2 \times \frac{65{,}815^2}{142} + 3852^2 \times \frac{53{,}489^2}{68}\right)} = 53{,}983{,}055$$

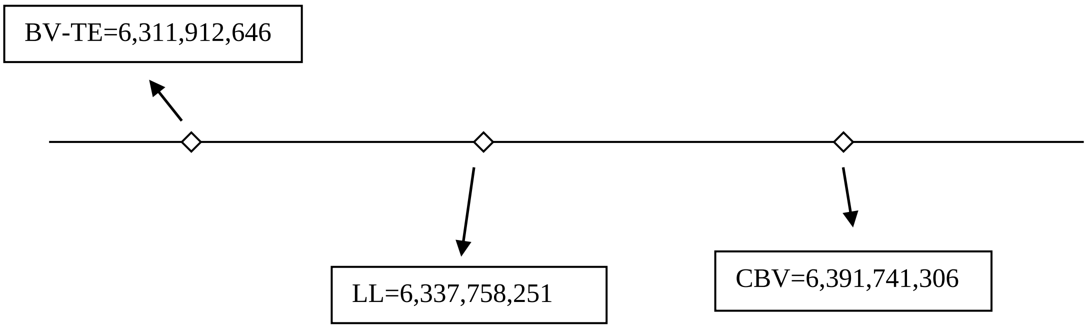
To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE = 6{,}391{,}741{,}306 - 53{,}983{,}055 = 6{,}337{,}758{,}251$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE)

$$BV - TE = 6{,}440{,}727{,}190 - 128{,}814{,}544 = 6{,}311{,}912{,}646$$

Finally, since $BV - TE$ is lower than the lower limit $LL = CBV - SE$ than we can conclude there is enough evidence that errors in the programme are smaller than materiality threshold.



7.3 Monetary unit sampling

7.3.1 Standard approach

7.3.1.1 Introduction

Monetary unit sampling is the statistical sampling method that uses the monetary unit as an auxiliary variable for sampling. This approach is usually based on systematic sampling with probability proportional to size (PPS), i.e. proportional to the monetary value of the sampling unit (higher value items have higher probability of selection).

This is probably the most popular sampling method for auditing and is particularly useful if book values have high variability and there is positive correlation (association) between errors and book values. In other words, whenever it is expected that items with higher values tend to exhibit higher errors, situation that frequently holds in the audit framework.

Whenever the above conditions hold, i.e. book values have high variability and error are positively correlated (associated) with book values, then MUS tends to produce smaller sample sizes than equal probability based methods, for the same level of precision.

It should also be noted that samples produced by this method will typically have an over representation of high value items and an under representation of low value items. This is not a problem by itself as the method accommodates this fact in the extrapolation process, but makes sample results (e.g. sample error rate) as non-interpretable (only extrapolated results can be interpreted).

As equal probability based methods, this method can be combined with stratification (favourable conditions for stratification are discussed in Section 6.2 and specific formulas are presented in Section 7.3.2).

7.3.1.2 Sample size

Computing sample size n within the framework of monetary unit sampling relies on the following information:

- Population book value (total declared expenditure) BV
- Confidence level determined from systems audit and the related coefficient z from a normal distribution (see Section 6.3)
- Maximum tolerable error TE (usually 2% of the total expenditure)
- Anticipated error AE chosen by the auditor according to professional judgment and previous information
- The standard deviation σ_r of the error rates (produced from a MUS sample).

The sample size is computed as follows:

$$n = \left(\frac{z \quad \times BV \quad \times \sigma_r}{TE - AE} \right)^2$$

where σ_r is the standard-deviation of error rates produced from a MUS sample. To obtain an approximation to this standard-deviation before performing the audit the Member States will have to rely either on historical knowledge (variance of the error rates in a sample of past period) or on a preliminary/pilot sample of low sample size, n^p (sample size for the preliminary sample is recommended to be not less than 20 to 30 operations). In any case, the variance of the error rates (square of the standard-deviation) is obtained through

$$\sigma_r^2 = \frac{1}{n^p - 1} \sum_{i=1}^{n^p} (r_i - \bar{r})^2 \,;$$

where $r_i = \frac{E_i}{BV_i}$ is the error rate of an operation and is defined as the ratio between E_i and the book value (the expenditure certified to the Commission, BV_i) of the i-th operation included in sample and $\bar{r}$ represent the mean error rate in the sample, that is[18]:

$$\bar{r} = \frac{1}{n^p} \sum_{i=1}^{n^p} \frac{E_i}{BV_i}$$

As usual, if the standard-deviation is based on a preliminary sample, this sample can be subsequently used as a part of the full sample chosen for audit. Nevertheless, selecting and observing a preliminary sample in MUS framework is a much more complex task than in simple random sampling or difference estimation. This is because high value

[18] Whenever the book value of unit i (BV_i) is larger than the cut-off BV/n^p the ratio $\frac{E_i}{BV_i}$ should be substituted by $\frac{E_i}{BV/n^p}$ in the ratios.

items are more frequently chosen to the sample. Therefore, observing a 20 to 30 operations sample will frequently constitute a heavy task. Due to this reason, in the framework of MUS it is highly recommended that the estimation of the standard-deviation σ_r is based on historical data, in order to avoid the need to select a preliminary sample.

7.3.1.3 Sample selection

After determining sample size it is necessary to identify the high value population units (if any) that will belong to a high value stratum to be audited a 100%. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV) and the planed sample size (n). All items whose book value is higher than this cut-off (if $BV_i > BV/n$) will be placed in the 100% audit stratum.

The sampling size to be allocated to the non-exhaustive stratum, n_s , is computed as the difference between n and the number of sampling units (e.g. operations) in the exhaustive stratum (n_e).

Finally the selection of the sample in the non-exhaustive stratum will be made using probability proportional to size, i.e. proportional to the item book values BV_i[19]. A popular way to implement the selection is through systematic selection, using a sampling interval equal to the total expenditure in the non-exhaustive stratum (BV_s) divided by the sample size (n_s), i.e.

$$SI = \frac{BV_s}{n_s}$$

In practice the sample is selected from a randomised list of items (usually operations), selecting each item containing the xth monetary unit, x being equal to the sampling interval and having a random starting point between 1 and SI. For instance, if a population has a book value of 10,000,000€, and we select a sample of 40 operations, every operation containing the 250,000th€ will be selected.

7.3.1.4 Projected error

The projection of the errors to the population should be made differently for the units in the exhaustive stratum and for the items in the non-exhaustive stratum.

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[19] This can be performed using specialized software, any statistical package or even a basic software as Excel. Note that in some software the division between the exhaustive high value stratum and the non-exhaustive stratum is not necessary as they automatically accommodate the selection of units with a 100% selection probability.

For the exhaustive stratum, that is, for the stratum containing the sampling units with book value larger than the cut-off, $BV_i > \frac{BV}{n}$, the projected error is just the summation of the errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the non-exhaustive stratum, i.e. the stratum containing the sampling units with book value smaller or equal to the cut-off value, $BV_i \leq \frac{BV}{n}$ the projected error is

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

To calculate this projected error:
1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$
2) sum these error rates over all units in the sample
3) multiply the previous result by the total expenditure in the population of the non-exhaustive stratum (BV_s); this expenditure will also be equal to the total expenditure in the population minus the expenditure of items belonging to the exhaustive stratum
4) divide the previous result by the sample size in the non-exhaustive stratum (n_s)

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

7.3.1.5 Precision

Precision is a measure of the uncertainty associated with the extrapolation. It represents sampling error and should be calculated in order to subsequently produce a confidence interval.

The precision is given by the formula:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r$$

where s_r is the standard-deviation of error rates in the sample of the non-exhaustive stratum (calculated from the same sample used to extrapolate the errors to the population)

$$s_r^2 = \frac{1}{n_s - 1} \sum_{i=1}^{n_s} (r_i - \bar{r}_s)^2$$

having $\bar{r}_s$ equal to the simple average of error rates in the sample of the stratum

$$\bar{r}_s = \frac{\sum_{i=1}^{n_s} \frac{E_i}{BV_i}}{n_s}$$

Note that the sampling error is only computed for the non-exhaustive stratum, since there is no sampling error to account for in the exhaustive stratum.

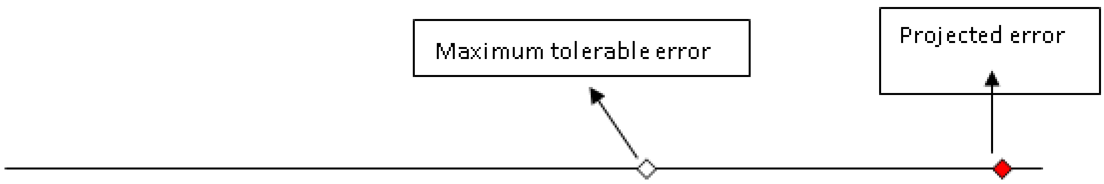
7.3.1.6 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

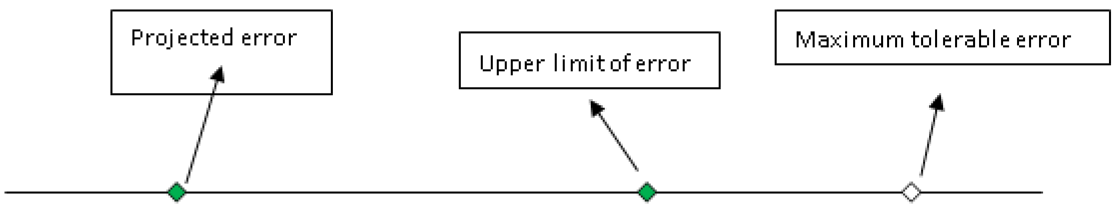
$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

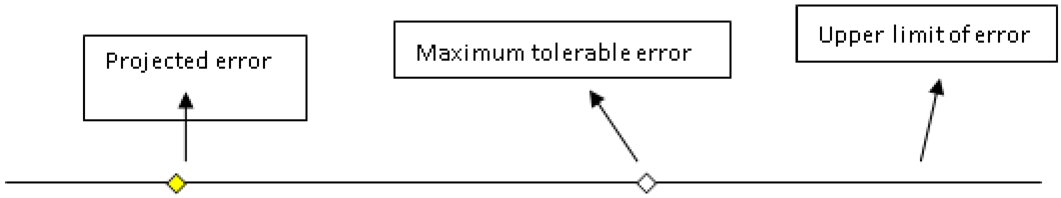
- If projected error is larger than maximum tolerable error, it means that the auditor would conclude that there is enough evidence to support that errors in the population are larger than materiality threshold:



- If the upper limit of error is lower than maximum tolerable error, then the auditor should conclude that errors in the population are lower than materiality threshold.



- If the projected error is lower than maximum tolerable error but the upper limit of error is larger than it means that additional work is needed as there is not enough evidence to support that the population is not materially misstated. The specific additional work needed is discussed in Section 5.11.



7.3.1.7 Example

Let's assume a population of expenditure certified to the Commission in a given year for operations in a programme. The system audits performed by the audit authority have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%.

The population is summarised in the following table:

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference year)	4,199,882,024 €

The sample size is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_r}{TE - AE}\right)^2$$

where σ_r is the standard-deviation of error rates produced from a MUS sample. To obtain an approximation to this standard deviation the AA decided to use the standard deviation of previous year. The sample of the previous year was constituted by 50 operations, 5 of which have a book value larger than the sampling interval.

The following table shows the results of the previous year's audit for these 5 operations.

Operation ID	Book Value (BV)	Correct Book Value (CBV)	Error	Error rate
1850	115,382,867 €	115,382,867 €	- €	-

4327	129,228,811 €	129,228,811 €	- €	-
4390	142,151,692 €	138,029,293 €	4,122,399 €	0.0491
1065	93,647,323 €	93,647,323 €	- €	-
1817	103,948,529 €	100,830,073 €	3,118,456 €	0.0371

Notice that the error rate (last column) is computed as $r_i = \frac{E_i}{BV/n}$ the ratio between the error of the operation and the BV divided by the initial sample size, that is 50, because these operations have a book value larger than the sampling interval (for more details please check Section 7.3.1.2).

The following tables summarises the results of last year's audit for the sample of 45 operations with the book value smaller than the cut-off value.

	A	B	C	D	E
1	**Operation ID**	**Book Value (BV)**	**Audit Value (AV)**	**Error**	**Error rate**
2	239	10,173,875 €	9,962,918 €	210,956 €	0.0207
3	424	23,014,045 €	23,014,045 €	- €	
4	2327	32,886,198 €	32,886,198 €	- €	
5	5009	34,595,201 €	34,595,201 €	- €	
6	1491	78,695,230 €	78,695,230 €	- €	
7	(…)	(…)	(…)	(…)	(…)
39	2596	8,912,999 €	8,909,491 €	3,508 €	0.00039
40	779	26,009,790 €	26,009,790 €	- €	-
41	1250	264,950 €	264,950 €	- €	-
42	3895	30,949,004 €	30,949,004 €	- €	-
43	2011	617,668 €	617,668 €	- €	-
44	4796	335,916 €	335,916 €	- €	-
45	3632	7,971,113 €	7,971,113 €	- €	-
46	2451	17,470,048 €	17,470,048 €	- €	-
47	**Sample standard deviation:=STDEV(E2:E46;0;0;0.0491;0;0.0371)**-------------->				0.085

Based on this preliminary sample the standard deviation of the error rates, σ_r , is 0.085, (computed in MS Excel as **":=STDEV(E2:E46;0;0;0.0491;0;0.0371)**")

Given this estimate for the standard deviation of error rates, the maximum tolerable error and the anticipated error, we are in conditions to compute the sample size. Assuming a tolerable error which is 2% of the total book value, 2% x 4,199,882,024 = 83,997,640, (materiality value set by the regulation) and an anticipated error rate of 0.4%, 0.4% x 4,199,882,024 = 16,799,528 (which corresponds to strong belief of the AA based both on past year's information and the results of the report on assessment of management and control systems),

$$n = \left(\frac{1.645 \times 4{,}199{,}882{,}024 \times 0.085}{83{,}997{,}640 - 16{,}799{,}528}\right)^2 \approx 77$$

In first place, it is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted at a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV) and the planed sample size (n). All items whose book value is higher than this cut-off (if $BV_i > BV/n$) will be placed in the 100% audit stratum. In this case the cut-off value is 4,199,882,024/77=54,593,922 €,.

The AA put in an isolated stratum all the operations with book value larger than 54,593,922, which corresponds to 8 operations, amounting to 786,837,081 €

The sampling interval for the remaining population is equal to the book value in the non-exhaustive stratum (BV_s) (the difference between the total book value and the book value of the eight operations belonging to the top stratum) divided by the number of operations to be selected (77 minus the 8 operations in the top stratum).

$$Sampling\ interval = \frac{BV_s}{n_s} = \frac{4{,}199{,}882{,}024 - 786{,}837{,}081}{69} = 49{,}464{,}419$$

The sample is selected from a randomised list of operations, selecting each item containing the 49,464,419th monetary unit.

A file containing the remaining 3,844 operations (3,852 – 8 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample value of 69 operations (77 minus 8 high value operations) is drawn using exactly the following procedure.

A random value between 1 and the sampling interval, 49,464,419 has been generated (22,006,651). The first selection corresponds to the first operation in the file with the accumulated book value greater or equal to 22,006,651.

The second selection corresponds to the first operation containing the 71,471,070th monetary unit ($22{,}006{,}651 + 49{,}464{,}419 = 71{,}471{,}070$ starting point plus the sampling interval). The third operation to be selected corresponds to the first operation containing the 120,935,489th monetary unit ($71{,}471{,}070 + 49{,}464{,}419 = 120{,}935{,}489$ previous monetary unit point plus the sampling interval) and so on…

Operation ID	Book Value (BV)	AcumBV	Sample
239	10,173,875 €	10,173,875 €	No
424	23,014,045 €	33,187,920 €	Yes
2327	32,886,198 €	66,074,118 €	No
5009	34,595,201 €	100,669,319 €	Yes

1491	78,695,230 €	179,364,549 €	Yes
(…)	(…)	(…)	…
2596	8,912,999 €	307,654,321 €	No
779	26,009,790 €	333,664,111 €	Yes
1250	264,950 €	333,929,061 €	No
3895	30,949,004 €	364,878,065 €	No
2011	617,668 €	365,495,733 €	No
4796	335,916 €	365,831,649 €	No
3632	7,971,113 €	373,802,762 €	Yes
2451	17,470,048 €	391,272,810 €	No
(…)	(…)	(…)	…

After auditing the 77 operations, the AA is able to project the error.

Out of the 8 high-value operations (total book value of 786,837,081 €), 3 operations contain error corresponding to an amount of error of 7,616,805 €.

For the remaining sample, the error has a different treatment. For these operations, we follow the following procedure:

1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$

2) sum these error rates over all units in the sample (computed in MS Excel as **":=SUM(E2:E70)**")

3) multiply the previous result by the sampling interval (SI)

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

	A	B	C	D	E
1	**Operation ID**	**Book Value (BV)**	**Audited Value (AV)**	**Error**	**Error rate**
2	5002	48,725,645 €	48,725,645 €	- €	-
3	779	26,009,790 €	333,664,111 €	- €	-
4	2073	859,992 €	859,992 €	- €	-
5	239	10,173,875 €	9,962,918 €	210,956 €	0.02
6	989	394,316 €	394,316 €	- €	-
7	65	25,234,699 €	25,125,915 €	108,784 €	0
8	5010	34,595,201 €	34,595,201 €	- €	-
9	(...)	(...)	(...)	(...)	(...)
64	1841	768,278 €	768,278 €	- €	-
65	3672	624,882 €	624,882 €	- €	-
66	2355	343,462 €	301,886 €	41,576 €	0.12
67	959	204,847 €	204,847 €	- €	-
68	608	15,293,716 €	15,293,716 €	- €	-
69	4124	6,773,014 €	6,773,014 €	- €	-
70	262	662 €	662 €	- €	-
71	**Total:=SUM(E2:E70)**--				1.096
72	**Sample standard deviation:=STDEV(E2:E70)**-------------------------------->				0.09

$$EE_s = 49{,}464{,}419 \times 1.096 = 54{,}213{,}004$$

The projected error at the level of population is just the sum of these two components:

$$EE = 7{,}616{,}805 + 54{,}213{,}004 = 61{,}829{,}809$$

The projected error rate is the ratio between the projected error and the total expenditure:

$$r = \frac{61{,}829{,}809}{4{,}199{,}882{,}024} = 1.47\%$$

The standard deviation of error rates in the sampling stratum is 0.09 (computed in MS Excel as **":=STDEV(E2:E70)**").

The precision is given by:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r = 1.645 \times \frac{4{,}199{,}882{,}024 - 786{,}837{,}081}{\sqrt{69}} \times 0.09 = 60{,}831{,}129$$

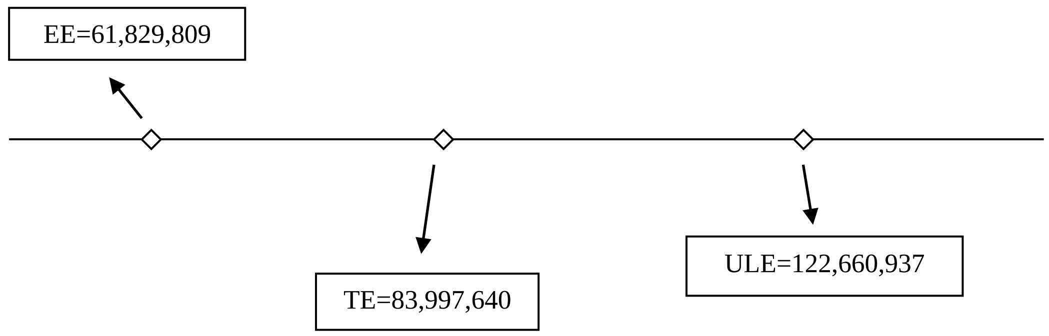
Note that the sampling error is computed for the non-exhaustive stratum only, since there is no sampling error to account for in the exhaustive stratum.

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = 61{,}829{,}809 + 60{,}831{,}129 = 122{,}660{,}937$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error, 83,997,640 €, to draw audit conclusions.

Since the maximum tolerable error is larger than the projected error, but smaller than the upper limit of error, then additional work is needed to support that the population is not materially misstated. The specific additional work needed is discussed in section 5.11.



7.3.2 Stratified monetary unit sampling

7.3.2.1 Introduction

In stratified monetary unit sampling, the population is divided in sub-populations called strata and independent samples are drawn from each stratum, using the standard monetary unit sampling approach.

As usual, candidate criteria to implement stratification should take into account that in stratification we aim to find groups (strata) with less variability than the whole population. Therefore, any variables that we expect to explain the level of error in the operations are also good candidates for stratification. Some possible choices are programmes, regions, responsible bodies, classes based on the risk of the operation, etc. Note that contrarily to what happens in equal probability sampling methods, in stratified MUS, the stratification by level of expenditure is not interesting, as MUS already takes into account the level of expenditure in the selection of sampling units.

7.3.2.2 Sample size

The sample size is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates for the whole set of strata, with the weight for each stratum equal to the ratio between the stratum book value (BV_h) and the book value for the whole population (BV).

$$\sigma_{rw}^2 = \sum_{i=1}^{H} \frac{BV_h}{BV} \sigma_{rh}^2, h = 1,2, \dots , H;$$

and σ_{rh}^2 is the variance of error rates in each stratum. The variance of the errors rates is computed for each stratum as an independent population as

$$\sigma_{rh}^2 = \frac{1}{n_h^p - 1} \sum_{i=1}^{n_h^p} (r_{hi} - \bar{r}_h)^2 , h = 1,2, \dots , H$$

where $r_{hi} = \frac{E_i}{BV_i}$ represent the individual error rates for units in the sample of stratum *h* and $\bar{r}_h$ represent the mean error rate of the sample in stratum *h*[20].

As previously presented for the standard MUS method these values can be based on historical knowledge or on a preliminary/pilot sample of low sample size. In this later case the pilot sample can as usual subsequently be used as a part of the sample chosen for audit. The recommendation of calculating these parameters using historical data again holds, in order to avoid the need to select a preliminary sample. When starting applying the stratified MUS method for the first time, it may happen that historical stratified data is unavailable. In this case, sample size can be determined using the formulas for the standard MUS method (see Section 7.3.1.2). Obviously the price to by this lack of historical knowledge is that on the first period of audit the sample size will be larger than in fact would be needed if that information were available. Nevertheless, the information collected in the first period of application of the stratified MUS method can be applied in future periods for sample size determination.

Once the total sample size, n, is computed the allocation of the sample by stratum is as follows:

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[20] Whenever the book value of unit i (BV_i) is larger than the cut-off BV_h / n_h the ratio $\frac{E_i}{BV_i}$ should be substituted by the ratios $\frac{E_i}{BV_h/n_h}$.

$$n_h = \frac{BV_h}{BV} n.$$

This is a general allocation method, where the sample is allocated to strata proportionally to the expenditure (book value) of the strata. Other allocation methods are available. A more tailored allocation may in some cases bring additional precision gains or reduction of sample size. The adequacy of other allocation methods to each specific population requires some technical knowledge in sampling theory.

7.3.2.3 Sample selection

In each stratum h, there will be two components: the exhaustive group inside stratum h (that is, the group containing the sampling units with book value larger than the cut-off value, $BV_{hi} > \frac{BV_h}{n_h}$); and the sampling group inside stratum h (that is, the group containing the sampling units with book value smaller or equal than the cut-off value, $BV_{hi} \leq \frac{BV_h}{n_h}$)

After determining sample size, it is necessary to identify in each of the original stratum (h) the high value population units (if any) that will belong to a high value group to be audited a 100%. The cut-off value for determining this top group is equal to the ratio between the book value of the stratum (BV_h) and the planed sample size (n_h). All items whose book value is higher than this cut-off (if $BV_{hi} > \frac{BV_h}{n_h}$) will be placed in the 100% audit group.

The sampling size to be allocated to the non-exhaustive group, n_{hs} , is computed as the difference between n_h and the number of sampling units (e.g. operations) in the exhaustive group of the stratum (n_{he}).

Finally the selection of the samples is done in the non-exhaustive group of each stratum using probability proportional to size, i.e. proportional to the item book values BV_i. A common way to implement the selection is through systematic selection, using a selection interval equal to the total expenditure in the non-exhaustive group of the stratum (BV_{hs}) divided by the sample size (n_{hs}), i.e.

$$SI_h = \frac{BV_{hs}}{n_{hs}}$$

Note that several independent samples will be selected, one for each original strata.

7.3.2.4 Projected error

The projection of errors to the population is made differently for units belonging to the exhaustive groups and for items in the non-exhaustive groups.

For the exhaustive groups, that is, for the groups containing the sampling units with book value larger than the cut-off value, $BV_{hi} > \frac{BV_h}{n_h}$, the projected error is the summation of the errors found in the items belonging to those groups:

$$EE_e = \sum_{h=1}^{H} \sum_{i=1}^{n_h} E_{hi}$$

In practice:
1) For each stratum h, identify the units belonging to the exhaustive group and sum their errors
2) Sum the previous results over the all set of *H* strata.

For the non-exhaustive groups, i.e. the groups containing the sampling units with book value lower or equal to the cut-off value, $BV_{hi} \leq \frac{BV_h}{n_h}$, the projected error is

$$EE_s = \sum_{h=1}^{H} \frac{BV_{sh}}{n_{sh}} \sum_{i=1}^{n_{sh}} \frac{E_{hi}}{BV_{hi}}$$

To calculate this projected error:
1) in each stratum h, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{hi}}{BV_{hi}}$
2) in each stratum h, sum these error rates over all units in the sample
3) in each stratum h, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{sh}); this expenditure will also be equal to the total expenditure in the stratum minus the expenditure of items belonging to the exhaustive group
4) in each stratum h, divide the previous result by the sample size in the non-exhaustive group (n_{sh})
5) sum the previous results over the whole set of *H* strata

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

7.3.2.5 Precision

As for the standard MUS method, precision is a measure of the uncertainty associated with the extrapolation. It represents sampling error and should be calculated in order to subsequently produce a confidence interval.

The precision is given by the formula:

$$SE = z \times \sqrt{\sum_{h=1}^{H} \frac{BV_{sh}^2}{n_{sh}} . s_{rsh}^2}$$

where s_{rsh} is the standard-deviation of error rates in the sample of the non-exhaustive group of stratum h (calculated from the same sample used to extrapolate the errors to the population)

$$s_{rsh}^2 = \frac{1}{n_{sh} - 1} \sum_{i=1}^{n_{sh}} (r_{hi} - \bar{r}_{sh})^2 \, , h = 1,2, \dots, H$$

having $\bar{r}_{sh}$ equal to the simple average of error rates in the sample of the non-exhaustive group of stratum h.

The sampling error is only computed for the non-exhaustive groups, since there is no sampling error arising from the exhaustive groups.

7.3.2.6 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 7.3.1.6.

7.3.2.7 Example

Assuming a population as expenditure certified to the Commission in a given year for operations in a group of two programmes. The system audits performed by the AA have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%.

The AA has reasons to believe that there are different error rates across the programmes. Bearing in mind all this information, the audit authority decided to stratify the population by programme.

The following table summarizes the available information.

Population size (number of operations)	6,252
Population size – stratum 1	4,520
Population size – stratum 2	1,732
Book value (sum of the expenditure in the reference year)	4,199,882,024 €
Book value – stratum 1	2,506,626,292 €
Book value – stratum 2	1,693,255,732 €

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates for the whole set of strata, with the weight for each stratum equal to the ratio between the stratum book value (BV_h) and the book value for the whole population (BV):

$$\sigma_{rw}^2 = \sum_{i=1}^{H} \frac{BV_h}{BV} \sigma_{rh}^2, h = 1,2, \dots, H;$$

where σ_{rh} is the standard deviation of error rates produced from a MUS sample. To obtain an approximation to this standard deviation the AA decided to use the standard deviation of previous year. The sample of the previous year was constituted by 110 operations, 70 operations from the first programme (stratum) and 40 from the second programme.

Based on this last year's sample we calculate the variance of the error rates as (see Section 7.3.1.7 for details):

$$\sigma_{r1}^2 = \frac{1}{70-1}\sum_{\text{i=1}}^{70}(r_{1i} - \bar{r}_{s1})^2 = 0.000045$$

and

$$\sigma_{r2}^2 = \frac{1}{40-1}\sum_{\text{i=1}}^{40}(r_{2i} - \bar{r}_{s2})^2 = 0.010909$$

This leads to the following result

$$\sigma_{rw}^2 = \frac{2{,}506{,}626{,}292}{4{,}199{,}882{,}024} \times 0.000045 + \frac{1{,}693{,}255{,}732}{4{,}199{,}882{,}024} \times 0.010909 = 0.004425$$

Given this estimate for the variance of error rates we are in conditions to compute the sample size. As already stated the AA expects significant differences across both strata. Further, based on report on the functioning of the management and control system, the audit authority expects an error rate around 1.1%. Assuming a tolerable error which is 2% of the total book value (materiality level set by the Regulation), that is, *TE*=2% x 4,199,882,024=83,997,640, and the anticipated error, i.e., AE=1.1% x 4,199,882,024=46,198,702, the sample size is

$$n = \left(\frac{1.645 \times 4{,}199{,}882{,}024 \times \sqrt{0.004425}}{83{,}997{,}640 - 46{,}198{,}702}\right)^2 \approx 148$$

The allocation of the sample by stratum is as follows:

$$n_1 = \frac{BV_1}{BV} \times n = \frac{2{,}506{,}626{,}292}{4{,}199{,}882{,}024} \times 148 = 89$$

$$n_2 = n - n_1 = 148 - 88 = 59.$$

These two samples sizes lead to the following values of cut-off for high-value strata:

$$Cut - off_1 = \frac{BV_1}{n_1} = \frac{2{,}506{,}626{,}292}{89} = 28{,}164{,}340$$

and

$$Cut - off_2 = \frac{BV_2}{n_2} = \frac{1{,}693{,}255{,}731}{59} = 28{,}699{,}250$$

Using these two cut-off values, 16 and 12 high value operations are found in stratum 1 and stratum 2, respectively.

The sample size for the sampling part of stratum 1 will be given by total sample size (89), deducted from the 16 high-value operations, i.e., 73 operations. Applying the same reasoning for stratum 2, the sample size for the sampling part of stratum 2 is 59-12=47 operations.

The next step will be the calculation of sampling interval for the sampling strata. The sampling intervals are, respectively, given by:

$$SI_1 = \frac{BV_{s1}}{n_{s1}} = \frac{1{,}643{,}963{,}924}{73} = 22{,}520{,}054$$

and

$$SI_2 = \frac{BV_{s2}}{n_{s2}} = \frac{1{,}059{,}467{,}667}{47} = 22{,}541{,}865$$

The following table summarises the previous results:

Population size (number of operations)	6,252
Population size – stratum 1	4,520
Population size – stratum 2	1,732
Book value (sum of the expenditure in the reference year)	4,199,882,024 €
Book value – stratum 1	2,506,626,292 €
Book value – stratum 2	1,693,255,731 €
Sample results – stratum 1	
Cut-off value	28,164,340 €
Number of operations above cut-off value	16
Book value of operations above cut-off value	862,662,369 €
Book value of operations (non-exhaustive population)	1,643,963,923 €
Sampling interval (non-exhaustive population)	22,520,054 €
Number of operations (non-exhaustive population)	4,504
Sample results – stratum 2	
Cut-off value	28,699,250 €
Number of operations above cut-off value	12
Book value of operations above cut-off value	633,788,064 €
Book value of operations (non-exhaustive population)	1,059,467,668 €
Sampling interval (non-exhaustive population)	22,541,865 €
Number of operations (non-exhaustive population)	1,720

For stratum 1, a file containing the remaining 4,504 operations (4,520 minus 16 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample of 73 operations (89 minus 16 high value operations) is drawn using exactly the same procedure as described in Section 7.3.1.7.

For stratum 2, a file containing the remaining 1,720 operations (1,732 minus 12 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample value of 47 operations (59 minus 12 high value operations) is drawn as described in previous paragraph.

For stratum 1, in the 16 high-value operations no errors were found.

For stratum 2, in 6, out of the 12 high-value operations, errors that amount to 15,460,340 € were found.

For the remaining samples the error has a different treatment. For these operations we follow the following procedure:

1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$

2) sum these error rates over all units in the sample

3) multiply the previous result by the sampling interval (SI)

$$EE_{hs} = SI_{hs} \sum_{i=1}^{n_{hs}} \frac{E_{hi}}{BV_{hi}}$$

The sum of the error rates for the non-exhaustive population in stratum 1 is 1.0234,

$$EE_{1s} = 22{,}520{,}054 \times 1.0234 = 23{,}047{,}023$$

and for stratum 2 is 1.176,

$$EE_{2s} = 22{,}541{,}865 \times 1.176 = 26{,}509{,}234.$$

The projected error at the level of population is just the sum of all the components, that is, the amount of error found in the exhaustive part of both strata, which is 15,460,340 € and the projected error for both strata:

$$EE = 15{,}460{,}340 + 23{,}047{,}023 + 26{,}509{,}234 = 65{,}016{,}597$$

corresponding to a projected error rate of 1.55%.

To calculate the precision the variances of the error rates for both sampling strata have to be obtained using the same procedure as described in Section 7.3.1.7:

$$s_{1r}^2 = \frac{1}{72-1} \sum_{i=1}^{72} (r_{1i} - \bar{r}_{1s})^2 = 0.000036$$

and

$$s_{2r}^2 = \frac{1}{48-1}\sum_{i=1}^{48}(r_{2i} - \bar{r}_{2s})^2 = 0.0081$$

The precision is given by:

$$SE = z \times \sqrt{\sum_{h=1}^{H}\frac{BV_{sh}^2}{n_{sh}} \times s_{rsh}^2}$$

$$SE = 1.645 \times \sqrt{\frac{1{,}643{,}963{,}923^2}{73} \times 0.000036 + \frac{1{,}059{,}467{,}667^2}{47} \times 0.0081}$$
$$= 22{,}958{,}216$$

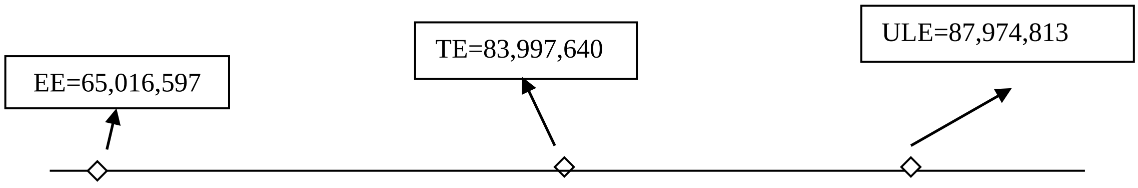
Note that the sampling error is only computed for the non-exhaustive parts of the population, since there is no sampling error to account for in the exhaustive stratum.

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = 65{,}016{,}597 + 22{,}958{,}216 = 87{,}974{,}813$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

Comparing to the materiality threshold of 2% of the total book value of the population (2% x 4,199,882,024 € = 83,997,640 €) with the projected results we observe that the maximum tolerable error is larger than the projected error, but smaller than the upper limit. Therefore, additional work (as described in Section 5.11) is needed as there is not enough evidence to support that the population is not materially misstated.



7.3.3 Monetary unit sampling – two periods

7.3.3.1 Introduction

The audit authority may decide to carry out the sampling process in several periods during the year (typically two semesters). As happens with all other sampling methods, the major advantage of this approach is not related with sample size reduction, but mainly allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

With this approach, the year population is divided in two sub-populations, each one corresponding to the operations and expenditure of each semester. Independent samples are drawn for each semester, using the standard monetary unit sampling approach.

7.3.3.2 Sample size

First semester
At the first period of auditing (e.g. semester) the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates in each semester, with the weight for each semester equal to the ratio between the semester book value (BV_t) and the book value for the whole population (BV).

$$\sigma_w^2 = \frac{BV_1}{BV}\sigma_{r1}^2 + \frac{BV_2}{BV}\sigma_{r2}^2$$

and σ_{rt}^2 is the variance of error rates in each semester. The variance of the errors rates is computed for each semester as

$$\sigma_{rt}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(r_{ti} - \bar{r}_t)^2\, , t = 1{,}2$$

where $r_{ti} = \frac{E_{ti}}{BV_{ti}}$ represent the individual error rates for units in the sample of semester t and $\bar{r}_t$ represent the mean error rate of the sample in semester t[21].

[21] Whenever the book value of unit i (BV_i) is larger than BV_t/n_t the ratio $\frac{E_{ti}}{BV_{ti}}$ should be substituted by the ratios $\frac{E_{ti}}{BV_t/n_t}$.

Values for the expected standard-deviations of error rates in both semesters have to be set using professional judgments and must be based on historical knowledge. The option to implement a preliminary/pilot sample of low sample size as previously presented for the standard monetary unit sampling method is still available, but can only be performed for the first semester. In fact, at the first moment of observation expenditure for the second semester has not yet taken place and no objective data (besides historical) is available. If pilot samples are implemented, they can, as usual, subsequently be used as a part of the sample chosen for audit.

If no historical data or knowledge is available to assess the variability of data in the second semester, a simplified approach can be used, computing the global sample size as

$$n = \left(\frac{z \times BV \times \sigma_{r1}}{TE - AE}\right)^2$$

Note, that in this simplified approach only information about the variability of error rates in the first period of observation is needed. The underlying assumption is that the variability of error rates will be of similar magnitude in both semesters.

Note that problems related to the lack of auxiliary historical information will usually be confined to the first year of the programming period. In fact, the information collected in the first year of auditing can be used in future year for sample size determination.

Also note that the formulas for sample size calculation require values for BV_1 and BV_2, i.e. total book value (declared expenditure) of the first and second semesters. When calculating sample size, the value for BV_1 will be known, but the value of BV_2 will be unknown and has to be imputed according to the expectations of the auditor (also based on historical information).

Once the total sample size, n, is computed the allocation of the sample by semester is as follows:

$$n_1 = \frac{BV_1}{BV} n$$

and

$$n_2 = \frac{BV_2}{BV} n$$

Second semester

At the first observation period, some assumptions were made relatively the following observation periods (typically the next semester). If characteristics of the population in the following periods differ significantly from the assumptions, sample size for the following period may have to be adjusted.

In fact, at the second period of auditing (e.g. semester) more information will be available:

- The total book value in the second semester BV_2 is correctly known;
- The sample standard-deviation of error rates s_{r1} calculated from the sample of the first semester is already available;
- The standard deviation of error rates for the second semester σ_{r2} can now be more accurately assessed using real data.

If these parameters are not dramatically different from the ones estimated at the first semester using the expectations of the auditor, the originally planned sample size, for the second semester (n_2), won't require any adjustments. Nevertheless, if the auditor considers that the initial expectations significantly differ from the real population characteristics, the sample size may have to be adjusted in order to account for these inaccurate estimates. In this case, the sample size of the second semester should be recalculated using

$$n_2 = \frac{\left(z \times BV_2 \times \sigma_{r2}\right)^2}{(TE - AE)^2 - z^2 \times \dfrac{BV_1^2}{n_1} \times s_{r1}^2}$$

where s_{r1} is the standard-deviation of error rates calculated from the sample of the first semester and σ_{r2} an estimate of the standard-deviation of error rates in the second semester based on historical knowledge (eventually adjusted by information from the first semester) or a preliminary/pilot sample of the second semester.

7.3.3.3 Sample selection

In each semester, the sample selection will exactly follow the procedure described for the standard monetary unit sampling approach. The procedure will be reproduced here for the sake of the reader.

For each semester, after determining sample size, it is necessary to identify the high value population units (if any) that will belong to a high value group to be audited a 100%. The cut-off value for determining this top group is equal to the ratio between the book value of the semester (BV_t) and the planed sample size (n_t). All items whose book value is higher than this cut-off (if $BV_{ti} > \frac{BV_t}{n_t}$) will be placed in the 100% audit group.

The sampling size to be allocated to the non-exhaustive group, n_{ts} , is computed as the difference between n_t and the number of sampling units (e.g. operations) in the exhaustive group (n_{te}).

Finally, in each semester, the selection of the samples is done in the non-exhaustive group using probability proportional to size, i.e. proportional to the item book values BV_{ti}. A popular way to implement the selection is though systematic selection, using a selection interval equal to the total expenditure in the non-exhaustive group (BV_{ts}) divided by the sample size (n_{ts}), i.e.

$$SI_h = \frac{BV_{ts}}{n_{ts}}$$

7.3.3.4 Projected error

The projection of errors to the population is calculated differently for units belonging to the exhaustive groups and for items in the non-exhaustive groups.

For the exhaustive groups, that is, for the groups containing the sampling units with book value larger than the cut-off value, $BV_{ti} > \frac{BV_t}{n_t}$, the projected error is the summation of the errors found in the items belonging to those groups:

$$EE_e = \sum_{i=1}^{n_1} E_{1i} + \sum_{i=1}^{n_2} E_{2i}$$

In practice:

1) For each semester *t*, identify the units belonging to the exhaustive group and sum their errors
2) Sum the previous results over the two semesters.

For the non-exhaustive groups, i.e. the groups containing the sampling units with book value lower or equal to the cut-off value, $BV_{ti} \leq \frac{BV_t}{n_t}$, the projected error is

$$EE_s = \frac{BV_{s1}}{n_{s1}} \times \sum_{i=1}^{n_{s1}} \frac{E_{1i}}{BV_{1i}} + \frac{BV_{s2}}{n_{s2}} \times \sum_{i=1}^{n_{s2}} \frac{E_{2i}}{BV_{2i}}$$

To calculate this projected error:

1) in each semester t, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{ti}}{BV_{ti}}$
2) in each semester t, sum these error rates over all units in the sample
3) in semester t, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{st}); this expenditure will also be equal to the total expenditure of the semester minus the expenditure of items belonging to the exhaustive group
4) in each semester t, divide the previous result by the sample size in the non-exhaustive group (n_{st})
5) sum the previous results over the two semesters

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

7.3.3.5 Precision

As for the standard MUS method, precision is a measure of the uncertainty associated with the extrapolation. It represents sampling error and should be calculated in order to subsequently produce a confidence interval.

The precision is given by the formula:

$$SE = z \times \sqrt{\frac{BV_{s1}^2}{n_{s1}} \times s_{rs1}^2 + \frac{BV_{s2}^2}{n_{s2}} \times s_{rs2}^2}$$

where s_{rs2} is the standard-deviation of error rates in the sample of the non-exhaustive group of semester t (calculated from the same sample used to extrapolate the errors to the population)

$$s_{rst}^2 = \frac{1}{n_{st} - 1} \sum_{i=1}^{n_{st}} (r_{ti} - \bar{r}_{st})^2 \, , t = 1,2$$

having $\bar{r}_{st}$ equal to the simple average of error rates in the sample of the non-exhaustive group of semester t.

The sampling error is only computed for the non-exhaustive groups, since there is no sampling error arising from the exhaustive groups.

7.3.3.6 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 7.3.1.6.

7.3.3.7 Example

In order to anticipate the audit workload that usually is concentrated at the end of the audit year the AA decided to spread the audit work in two periods. At the end of the first semester the AA considered the population divided into two groups corresponding to each one of the two semesters. At the end of the first semester the characteristics of the population are the following:

Declared expenditure at the end of first semester	1,827,930,259 €
Size of population (operations - first semester)	2,344

Based on the past experience, the AA knows that usually all the operations included in the programmes at the end of the reference period are already active in the population of the first semester. Moreover, it is expected that the declared expenditure at the end of the first semester represents about 35% of the total declared expenditure at the end of the reference year. Based on these assumptions a summary of the population is described in the following table:

Declared expenditure (DE) at the end of first semester	1,827,930,259 €
Declared expenditure (DE) at the end of the second semester (predicted) 1,827,930,259€ / 35%-1,827,930,259€) = 3,394,727,624€)	3,394,727,624 €
Total expenditure forecasted for the year	5.222.657.883€
Size of population (operations – first semester)	2,344
Size of population (operations – second semester, predicted)	2,344

For the first period, the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted average of the variances of the error rates in each semester, with the weight for each semester equal to the ratio between the semester book value (BV_t) and the book value for the whole population (BV).

$$\sigma_{rw}^2 = \frac{BV_1}{BV}\sigma_{r1}^2 + \frac{BV_2}{BV}\sigma_{r2}^2$$

and σ_{rt}^2 is the variance of error rates in each semester. The variance of the errors rates is computed for each semester as

$$\sigma_{rt}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(r_{ti} - \bar{r}_t)^2\,, t = 1,2, \dots, T$$

Since these variances are unknown, the AA decided to draw a preliminary sample of 20 operations at the end of first semester of the current year. The sample standard deviation of error rates in this preliminary sample at first semester is 0.12. Based on professional judgement and knowing that usually the expenditure in second semester is larger than in first semester, the AA has made a preliminary prediction of standard deviation of error rates for the second semester to be 110% larger than in first semester, that is, 0.25. Therefore, the weighted average of the variances of the error rates is:

$$\sigma_{rw}^2 = \frac{1{,}827{,}930{,}259}{1{,}827{,}930{,}259\ + 3{,}394{,}727{,}624} \times 0.12^2$$

$$+ \frac{3{,}394{,}727{,}624}{1{,}827{,}930{,}259\ + 3{,}394{,}727{,}624} \times 0.25^2 = 0.0457$$

In the first semester, the AA, given the level of functioning of the management and control system, considers adequate a confidence level of 60%. The global sample size for the whole year is:

$$n = \left(\frac{0.842 \times (1{,}827{,}930{,}259\ + 3{,}394{,}727{,}624) \times \sqrt{0.0457}}{104{,}453{,}158 - 20{,}890{,}632}\right)^2 \approx 127$$

where z is 0.842 (coefficient corresponding to a 60% confidence level), TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value. The total book value comprises the true book value at the end of the first semester plus the predicted book value for the second semester 3,394,727,624 €, which means that tolerable error is 2% x 5,222,657,883 € = 104,453,158 €. The last year's audit projected an error rate of 0.4%. Thus AE, the anticipated error, is 0.4% x 5,222,657,883 € = 20,890,632 €.

The allocation of the sample by semester is as follows:

$$n_1 = \frac{BV_1}{BV_1 + BV_2} = \frac{1{,}827{,}930{,}259}{1{,}827{,}930{,}259 \ + 3{,}394{,}727{,}624} \times 127 \approx 45$$

and

$$n_2 = n - n_1 = 83$$

For the first semester, it is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted at a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV_1) and the planned sample size (n_1). All items whose book value is higher than this cut-off (if $BV_{i1} > BV_1/n_1$) will be placed in the 100% audit stratum. In this case the cut-off value is 40,620,672 €. There are 11 operations which book value is larger than this cut-off value. The total book value of these operations amounts to 891,767,519 €.

The sampling size to be allocated to the non-exhaustive stratum (n_{s1}) is computed as the difference between n_1 and the number of sampling units in the exhaustive stratum (n_e), that is 34 operations.

The selection of the sample in the non-exhaustive stratum will be made using probability proportional to size, i.e. proportional to the item book values BV_{is1}, through systematic selection, using a sampling interval equal to the total expenditure in the non-exhaustive stratum (BV_{s1}) divided by the sample size (n_{s1}), i.e.

$$SI_{s1} = \frac{BV_{s1}}{n_{s1}} = \frac{1{,}827{,}930{,}259 - 891{,}767{,}519}{34} = 27{,}534{,}198$$

The book value in the non-exhaustive stratum (BV_{s1}) is just the difference between the total book value and the book value of the five operations belonging to the top stratum.

The following table summarises these results:

Cut-off value – first semester	40,620,672 €
Number of operations with book value larger than cut-off value - first semester	11
Book value of operations with book value larger than cut-off value - first semester	891,767,519 €
BV_{s1}- first semester	936,162,740 €
n_{s1}- first semester	34
SI_{s1}- first semester	27,534,198 €

Out of the 11 operations with book value larger than the sampling interval, 6 of them have error. The total error found in this stratum is 19,240,855 €.

A file containing the remaining 2,333 operations of the population is randomly sorted and a sequential cumulative book value variable is created. A sample of 34 operations is drawn using the systematic proportional to size procedure.
The value of the 34 operations is audited. The sum of the error rates for the first semester is:

$$\sum_{i=1}^{34} \frac{E_{is1}}{BV_{is1}} = 1.4256$$

The standard-deviation of error rates in the sample of the non-exhaustive population of the first semester is (see Section 7.3.1.7 for details):

$$s_{rs1} = \sqrt{\frac{1}{34-1}\sum_{i=1}^{34}(r_{is1} - \bar{r}_{s1})^2} = 0.085$$

having $\bar{r}_{s1}$ equal to the simple average of error rates in the sample of the non-exhaustive group of first semester.

At the end of the second semester more information is available, in particular, the total expenditure of operations active in the second semester is correctly known, the sample variance of error rates s_{r1} calculated from the sample of the first semester is already available and the standard deviation of error rates for the second semester σ_{r2} can now be more accurately assessed using a preliminary sample of real data.

The AA realises that the assumption made at the end of the first semester on the total expenditure, 3,394,727,624 €, overestimates the true value of 2,961,930,008. There are also two additional parameters for which updated figures should be used.

Firstly, the estimate of the standard deviation of error rates based on the first semester sample of 34 operations yielded an estimate of 0.085. This new value should now be used to reassess the planned sample size. Secondly, based on the increased expenditure of the second semester compared to the initial estimate, the AA considers more prudent to estimate the standard deviation of error rates for the second semester as 0.30 instead of the initial value of 0.25. The updated figures of standard deviation of error rates for both semesters are far from the initial estimates. As a result, the sample for the second semester should be revised.

Parameter	Forecast done in the first semester	End of second semester
Standard deviation of error rates in the first semester	0.12	0.085
Standard deviation of error rates in the second semester	0.25	0.30
Total expenditure in the second semester	3,394,727,624 €	2,961,930,008 €

Taking into consideration these three adjustments, the recalculated sample size of the second semester is

$$n_2 = \frac{\left(z \times BV_2 \ \times \sigma_{r2}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s_{r1}^2}$$

where s_{r1} is the standard-deviation of error rates calculated from the sample of the first semester (the sample also used to produce the projected error) and σ_{r2} an estimate of the standard-deviation of error rates in the second semester:

$$n_2 = \frac{(0.842 \times 2{,}961{,}930{,}008 \times 0.30)^2}{(95{,}797{,}205 - 19{,}159{,}441)^2 - 0.842^2 \times \frac{1{,}827{,}930{,}259^2}{45} \times 0.085^2} \approx 102$$

where:

- TE = (1,827,930,259€ + 2,961,930,008 €) * 2% = 95,797,205 €
- AE = (1,827,930,259€ + 2,961,930,008 €) * 0,4% = 19,159,441 €

It is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted at a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV_2) and the planed sample size (n_2). All items whose book value is higher than this cut-off (if $BV_{i2} > BV_2/n_2$) will be placed in the 100% audit stratum. In this case, the cut-off value is 29,038,529 €. There are 6 operations which book value is larger than this cut-off value. The total book value of these operations amounts to 415,238,983 €.

The sampling size to be allocated to the non-exhaustive stratum, n_{s2} , is computed as the difference between n_2 and the number of sampling units (e.g. operations) in the exhaustive stratum (n_{e2}), that is 96 operations (102, the sample size, minus the 6 high-value operations). Therefore, the auditor has to select in the sample using the sampling interval:

$$SI_{s2} = \frac{BV_{s2}}{n_{s2}} = \frac{2{,}961{,}930{,}008 - 415{,}238{,}983}{96} = 26{,}528{,}032$$

The book value in the non-exhaustive stratum (BV_{s2}) is just the difference between the total book value and the book value of the 6 operations belonging to the top stratum.

The following table summarises these results:

Cut-off value - second semester	29,038,529 €
Number of operations with book value larger than cut-off value - second semester	6
Book value of operations with book value larger than cut-off value- second semester	415,238,983 €
BV_{s2}- second semester	2,546,691,025 €
n_{s2}- second semester	96
SI_{s2}- second semester	26,528,032 €

Out of the 6 operations with book value larger than the cut-off value, 4 of them have error. The total error found in this stratum is 9,340,755 €.

A file containing the remaining 2,338 operations of the second semester population is randomly sorted and a sequential cumulative book value variable is created. A sample of 96 operations is drawn using the systematic proportional to size procedure.

The value of these 96 operations is audited. The sum of the error rates for the second semester is:

$$\sum_{i=1}^{96} \frac{E_{2i}}{BV_{2i}} = 1.1875$$

The standard-deviation of error rates in the sample of the non-exhaustive population of the second semester is:

$$s_{rs2} = \sqrt{\frac{1}{96-1}\sum_{i=1}^{96}(r_{is2} - \bar{r}_{s2})^2} = 0.29$$

having $\bar{r}_{s2}$ equal to the simple average of error rates in the sample of the non-exhaustive group of second semester.

The projection of errors to the population is made differently for units belonging to the exhaustive strata and for items in the non-exhaustive strata.

For the exhaustive strata, that is, for the strata containing the sampling units with book value larger than the cut-off, $BV_{ti} > \frac{BV_t}{n_t}$, the projected error is the summation of the errors found in the items belonging to those strata:

$$EE_e = \sum_{i=1}^{n_1} E_{1i} + \sum_{i=1}^{n_2} E_{2i} = 19{,}240{,}855 + 9{,}340{,}755 = 28{,}581{,}610$$

In practice:
1) For each semester *t*, identify the units belonging to the exhaustive group and sum their errors
2) Sum the previous results over the two semesters.

For the non-exhaustive group, i.e. the strata containing the sampling units with book value smaller or equal to the cut-off value, $BV_{ti} \le \frac{BV_t}{n_t}$, the projected error is

$$EE_s = \frac{BV_{s1}}{n_{s1}} \times \sum_{i=1}^{n_{s1}} \frac{E_{1i}}{BV_{1i}} + \frac{BV_{s2}}{n_{s2}} \times \sum_{i=1}^{n_{s2}} \frac{E_{2i}}{BV_{2i}}$$
$$= \frac{936{,}162{,}740}{34} \times 1.4256 + \frac{2{,}546{,}691{,}025}{96} \times 1.1875 = 70{,}754{,}790$$

To calculate this projected error:
1) in each semester *t*, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{ti}}{BV_{ti}}$
2) in each semester *t*, sum these error rates over all units in the sample
3) in semester *t*, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{st}); this expenditure will also be equal to the total expenditure of the semester minus the expenditure of items belonging to the exhaustive group
4) in each semester *t*, divide the previous result by the sample size in the non-exhaustive group (n_{st})
5) sum the previous results over the two semesters

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s = 28{,}581{,}610 + 70{,}754{,}790 = 99{,}336{,}400$$

corresponding to a projected error rate of 2.07%.

The precision is a measure of the uncertainty associated with the projection. The precision is given by the formula:

$$SE = z \times \sqrt{\frac{BV_{s1}^2}{n_{s1}} \times s_{rs1}^2 + \frac{BV_{s2}^2}{n_{s2}} \times s_{rs2}^2}$$
$$= 0.842 \times \sqrt{\frac{936{,}162{,}740^2}{34} \times 0.085^2 + \frac{2{,}546{,}691{,}025^{\,2}}{96} \times 0.29^2}$$
$$= 64{,}499{,}188$$

where s_{rst} are the standard-deviation of error rates already computed.

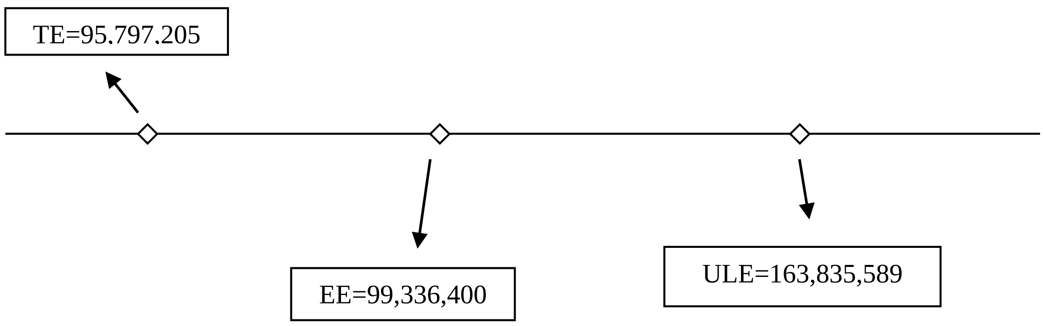
The sampling error is only computed for the non-exhaustive strata, since there is no sampling error arising from the exhaustive groups.

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the projection

$$ULE = EE + SE = 99{,}336{,}400 + 64{,}499{,}188 = 163{,}835{,}589$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions.

In this particular case, the projected error is larger than maximum tolerable error. It means that the auditor would conclude that there is enough evidence to support that errors in the population are larger than materiality threshold:



7.3.4 Conservative approach

7.3.4.1 Introduction

In the context of auditing it is usual to use a conservative approach to monetary unit sampling. This conservative approach has the advantage of requiring less knowledge about the population (for ex. no information about population variability is needed for sample size calculation). Also, several software packages used in the audit world automatically implement this approach turning easier its application. In fact, when adequately supported by these packages the application of the conservative method requires significantly less technical and statistical knowledge than the so-called standard approach. The main disadvantage of this conservative approach is in fact related with this easiness of application: as it uses less detailed information for sample size calculation and for precision determination it usually produces larger samples sizes and larger estimated sampling errors than the more exact formulas used in the standard approach. Nevertheless, whenever sample is already of a manageable size and not a major concern of the auditor, this approach can be a good option due to its simplicity.

This method cannot be combined with stratification or spreading the audit work in two or more periods within the reference year as it would result in unworkable formulas for precision determination. Therefore, the audit authorities are encouraged to use the standard approach for these purposes.

7.3.4.2 Sample size

The calculation of sample size n within the framework of monetary unit sampling conservative approach relies on the following information:

- Population book value (total declared expenditure) *BV*
- A constant called reliability factor (*RF*) determined by the confidence level
- Maximum tolerable error *TE* (usually 2% of the total expenditure)
- Anticipated error *AE* chosen by the auditor according to professional judgment and previous information
- The expansion factor, EF, which is a constant also associated with the confidence level and used when errors are expected

The sample size is computed as follows:

$$n = \frac{BV \times RF}{TE - (AE \times EF)}$$

The reliability factor RF is a constant from the Poisson distribution for an expected zero error. It is dependent on the confidence level and the values to apply in each situation can be found in the following table.

Confidence level	99%	95%	90%	85%	80%	75%	70%	60%	50%
Reliability Factor (RF)	4.61	3.00	2.31	1.90	1.61	1.39	1.21	0.92	0.70

Table 5. Reliability factors by confidence level

The expansion factor, $EF,$ is a factor used in the calculation of MUS sampling when errors are expected, which is based upon the risk of incorrect acceptance. It reduces the sampling error. If no errors are expected, the anticipated error (AE) will be zero and the expansion factor is not used. Values for the expansion factor are found in the following table.

Confidence level	99%	95%	90%	85%	80%	75%	70%	60%	50%
Expansion Factor (EF)	1.9	1.6	1.5	1.4	1.3	1.25	1.2	1.1	1.0

Table 6. Expansion factors by confidence level

The formula for sample size determination shows why this approach is called conservative. In fact sample size is neither dependent on the population size nor on the population variability. This means that the formula aims to fit any kind of population despite its specific characteristics, therefore usually producing sample sizes that are larger than the ones needed in practice.

7.3.4.3 Sample selection

After determining sample size, the selection of the sample is made using probability proportional to size, i.e. proportional to the item book values BV_i. A popular way to implement the selection is through systematic selection, using a sampling interval equal to the total expenditure (BV) divided by the sample size (n), i.e.

$$SI = \frac{BV}{n}$$

Typically, the sample is selected from a randomised list of all items, selecting each item containing the xth monetary unit, **x being the step corresponding to the book value divided by the sample size**, that is, the sampling interval.

Some items can be selected multiple times (if its value is above the size of the sampling interval). In this case, the auditor should create an exhaustive stratum where all the items with book value larger than the sampling interval should belong. This stratum will have a different treatment for error projection, as usual.

7.3.4.4 Projected error

The projection of the errors to the population follows the procedure presented in the context of the standard MUS approach. Again, the extrapolation is done differently for the units in the exhaustive stratum and for the items in the non-exhaustive stratum.

For the exhaustive stratum, that is, for the stratum containing the sampling units with book value larger than the sampling interval, $BV_i > \frac{BV}{n}$, the projected error is just the summation of the errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the non-exhaustive stratum, i.e. the stratum containing the sampling units with book value lower or equal to the sampling interval, $BV_i \leq \frac{BV}{n}$ the projected error is

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

To calculate this projected error:
1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$
2) sum these error rates over all units in the sample
3) multiply the previous result by the sampling interval (SI)

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

7.3.4.5 Precision

Precision, which is measuring sampling error, has two components: the Basic Precision, BP, and the Incremental allowance, IA.

The basic precision is just the product between sampling interval and the reliability factor (already used for calculating sample size):

$$BP = SI \times RF.$$

The incremental allowance is computed for every sampling unit belonging to the non-exhaustive stratum that contains an error.

Firstly, items with errors should be ordered by decreasing value of the error.

Secondly, an incremental allowance is calculated for each one of these items (with errors), using the formula:

$$IA_i = (RF(n) - RF(n-1) - 1) \times SI \times \frac{E_i}{BV_i}.$$

where $RF(n)$ is the reliability factor for the error that appear at n^{th} order at a given confidence level (typically the same used for sample size calculation), and $RF(n-1)$ is the reliability factor for the error at $(n-1)^{th}$ order at a given confidence level. For example, at 90% of confidence the corresponding table of reliability factors is:

Order of the error	**Reliability Factor (RF)**	$\boldsymbol{RF(n) - RF(n-1) - 1}$
Order zero	2.31	
1st	3.89	0.58
2nd	5.33	0.44
3rd	6.69	0.36
4th	8.00	0.31
…		

Table 7. Reliability factors by order of the error

For instance if the larger error in the sample is equal to 10,000€ (25% of the expenditure of 40,000€) and we have a sampling interval of 200,000€, the individual incremental allowance for this error is equal to 0.58 x 0.25 x 200,000=29,000€.

A table with reliability factors for several confidence levels and different number of errors found in the sample can be found in appendix.

Finally, the incremental allowance is the sum of all item incremental allowances:

$$IA = \sum_{i=1}^{n_s} IA_i.$$

The global precision (SE) will be equal to the sum of the two components: basic precision (BP) and incremental allowance (IA)

$$SE = BP + IA$$

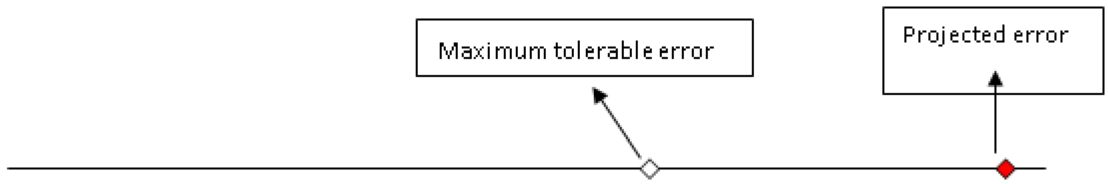
7.3.4.6 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the global precision of the extrapolation

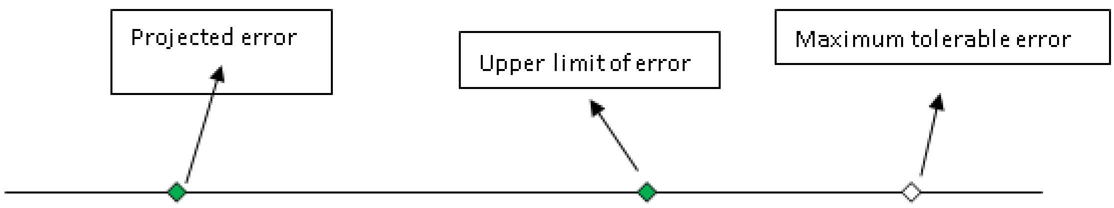
$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

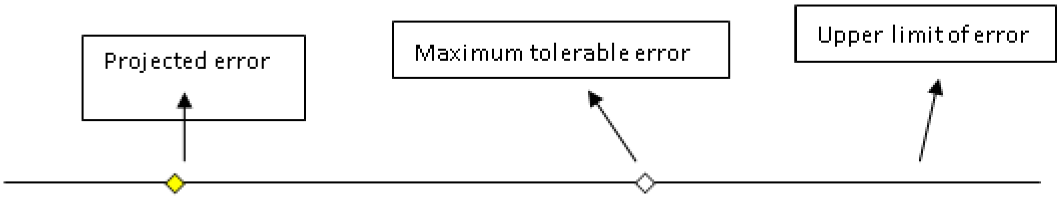
- If projected error is larger than maximum tolerable error, it means that the auditor would conclude that there is enough evidence to support that errors in the population are larger than materiality threshold:



- If the upper limit of error is lower than maximum tolerable error, then the auditor should conclude that errors in the population are lower than materiality threshold.



- If the projected error is lower than maximum tolerable error but the upper limit of error is larger than it means that additional work is needed as there is not enough evidence to support that the population is not materially misstated. The specific additional work needed is discussed in Section 5.11.



7.3.4.7 Example

Let's assume a population as expenditure certified to the Commission in a given year for operations in a programme. The system audits performed by the audit authority have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%.

The population is summarised in the table below:

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference year)	4,199,882,024 €

The sample size is computed as follows:

$$n = \frac{BV \times RF}{TE - (AE \times EF)},$$

where BV is the total book value of the population, that is, the total expenditure certified to the Commission in the reference year, RF is the reliability factor corresponding to the 90% confidence level, 2.31, $EF,$ is the expansion factor corresponding to the confidence level if errors are expected, 1.5. Regarding this particular population the audit authority, based on the past years' experience and on the knowledge of the improvements on the management and control system has decided that an expected error rate of 0.2% is reliable

$$n = \frac{4{,}199{,}882{,}024 \times 2.31}{0.02 \times 4{,}199{,}882{,}024 - (0.002 \times 4{,}199{,}882{,}024 \times 1.5)} \approx 136$$

The selection of the sample is made using probability proportional to size, i.e. proportional to the item book values, BV_i through systematic selection, using a sampling interval equal to the total expenditure (BV) divided by the sample size (n), i.e.

$$SI = \frac{BV}{n} = \frac{4{,}199{,}882{,}024}{136} = 30{,}881{,}485$$

A file containing the 3,852 operations of the population is randomly sorted and a sequential cumulative book value variable is created.

The sample is selected from this randomised list of all operations, selecting each item containing the 30,881,485th monetary unit.

Operation	**Book Value (BV)**	**AcumBV**
239	10,173,875 €	10,173,875 €
424	23,014,045 €	33,187,920 €
2327	32,886,198 €	66,074,118 €
5009	34,595,201 €	100,669,319 €
1491	78,695,230 €	179,364,549 €
(…)	**(…)**	**(…)**

A random value between 0 and the sampling interval, 30,881,485 is generated (16,385,476). The first item to be selected is the one that contains the 16,385,476th monetary unit. The second selection corresponds to the first operation in the file with the accumulated book value greater or equal to 16,385,476+30,881,485 and so on…

Operation	**Book Value (BV)**	**AcumBV**	**Sample**
239	10,173,875 €	10,173,875 €	No
424	23,014,045 €	33,187,920 €	Yes
2327	32,886,198 €	66,074,118 €	Yes
5009	34,595,201 €	100,669,319 €	Yes
1491	78,695,230 €	179,364,549 €	Yes
(…)	**(…)**	**(…)**	**(…)**
2596	8,912,999 €	307,654,321 €	Yes
779	26,009,790 €	333,664,111 €	No
1250	264,950 €	333,929,061 €	No
3895	30,949,004 €	364,878,065 €	Yes
2011	617,668 €	365,495,733 €	No
4796	335,916 €	365,831,649 €	No
3632	7,971,113 €	373,802,762 €	No
2451	17,470,048 €	391,272,810 €	Yes
(…)	**(…)**	**(…)**	**(…)**

There are 24 operations whose book value is larger than the sampling interval, meaning that each one is selected at least once (for instance, the operation 1491 is selected 3 times, cf. previous table). The book value of these 24 operations amounts to

1,375,130,377 €. Out of these 24 operations, 4 contain errors corresponding to an error amount of 7,843,574 €.

For the remaining sample the error have a different treatment. For these operations we use the following procedure:

1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$

2) sum these error rates over all units in the sample

3) multiply the previous result by the sampling interval (SI)

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

Operation	**Book Value (BV)**	**Correct Book Value (CBV)**	**Error**	**Error rate**
2596	8,912,999 €	8,912,999 €	- €	-
459	869,080 €	869,080 €	- €	-
2073	859,992 €	859,992 €	- €	-
239	10,173,875 €	9,962,918 €	210,956 €	0.02
989	394,316 €	394,316 €	- €	-
65	25,234,699 €	25,125,915 €	108,784 €	0.00
5010	34,595,201 €	34,595,201 €	- €	-
…	…	…	…	…
3632	7,971,113 €	7,971,113 €	- €	-
3672	624,882 €	624,882 €	- €	-
2355	343,462 €	301,886 €	41,576 €	0.12
959	204,847 €	204,847 €	- €	-
608	15,293,716 €	15,293,716 €	- €	-
4124	6,773,014 €	6,773,014 €	- €	-
262	662 €	662 €	- €	-
Total				1.077

$$EE_s = 30{,}881{,}485 \times 1.077 = 33{,}259{,}360$$

The projected error at the level of population is just the sum of these two components:

$$EE = 7{,}843{,}574 + 33{,}259{,}360 = 41{,}102{,}934$$

corresponding to a projected error rate of 0.98%.

In order to be able to build the upper limit of error one needs to calculate the two components of the precision, the Basic Precision, BP, and the Incremental allowance, IA.

The basic precision is just the product between sampling interval and the reliability factor (already used for calculating sample size):

$$BP = 30{,}881{,}485 \times 2.31 = 71{,}336{,}231$$

The incremental allowance is computed for every sampling unit belonging to the non-exhaustive stratum that contains an error.

Firstly, items with errors should be ordered by decreasing value of the error. Secondly, an incremental allowance is calculated for each one of these items (with errors), using the formula:

$$IA_i = (RF(n) - RF(n-1) - 1) \times SI \times \frac{E_i}{BV_i}.$$

where $RF(n)$ is the reliability factor for the error that appear at n^{th} order at a given confidence level (typically the same used for sample size calculation), and $RF(n-1)$ is the reliability factor for the error at $(n-1)^{th}$ order at a given confidence level (see table in the appendix).

Finally, the incremental allowance is the sum of all item incremental allowances:

$$IA = \sum_{i=1}^{n_s} IA_i.$$

The following table summarises these results for the 16 operations containing error:

Order	Error (A)	Error rate (B):=(A)/BV	Projected error:=(B)*SI	RF(n)	(RF(n)-RF(n-1))-1	IA_i
0				2.31		
1	4,705,321 €	0.212	6,546,875 €	3.89	0.58	3,797,187 €
…	…	…	…	…	…	…
12	26,952 €	0.001	29,488 €	18.21	0.29	8,552 €
13	12,332 €	0.024	741,156 €	19.50	0.30	218,916 €
14	7,706 €	0.012	370,578 €	20.80	0.30	109,458 €
15	6,822 €	0.020	617,630 €	22.09	0.30	182,430 €
16	4,787 €	0.008	247,052 €	23.39	0.30	72,972 €
Total		1.077	38,264,277 €			14,430,761 €

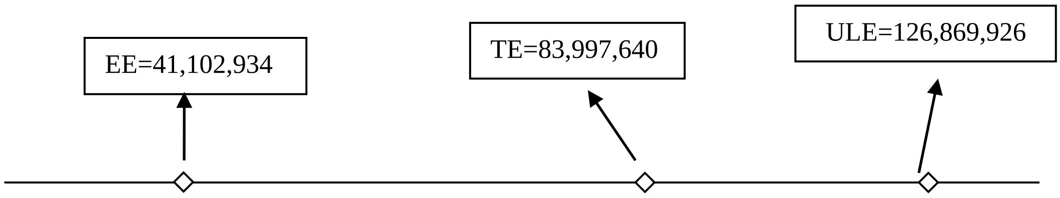
The global precision (SE) will be equal to the sum of the two components: basic precision (BP) and incremental allowance (IA)

$$SE = 71{,}336{,}231 + 14{,}430{,}761 = 85{,}766{,}992$$

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the global precision of the projection

$$ULE = 41{,}102{,}933 + 85{,}766{,}992 = 126{,}869{,}926$$

Now the maximum tolerable error, TE=2% x 4,199,882,024=83,997,640 € should be compared with both the projected error and the upper limit of error. The maximum tolerable error is larger than the projected error but smaller than the upper limit of error. Therefore, we conclude there is not sufficient evidence to find the population not materially misstated and additional work is needed as discussed in Section 5.11.



7.4 Non statistical sampling

7.4.1 Presentation

Remember (cf. Section 6.2) that non-statistical sampling should only be used:

- when having an extremely small population, whose size won't support the selection of a sample of adequate size (the population is smaller or very close to the recommended sample size).
- when it is not possible to observe the sample size that would be advisable for a statistical method, due to uncontrollable restrictions.

Non-statistical sampling should be avoided by all available means and should be reserved to situations where it is not possible to achieve a population with a sufficient size to apply statistical sampling. Consequently, when the auditor finds himself in the situation of having to apply non-statistical sampling, this means that sample size that would be advised by the application of appropriate formulas is not achievable. It is not possible to state the exact population size below which non-statistical sampling is needed as it depends on several population characteristics, but it is safe to state that this threshold is somewhere between 50 and 150 population units[22].

[22] Additionally remember the rule of thumb that settles the minimum sample size for statistical sampling equal to 30.

When implementing non statistical sampling two options may arise

- Option 1: If there are a few high-value operations in the population, a stratification by expenditure is advisable. For this stratification:
  - Determine the cut-off value of expenditure for items that will be included in the high value stratum; as a general rule the cut-off value is equal to the maximum tolerable error (2% of the total expenditure) of the population. This cut-off can and should be changed in accordance to population characteristics. For example, it will be typically enlarged for very small populations (below 50 units). The cut-off value should mainly be determined by professional judgments. Whenever the auditor can identify a few number of items whose expenditure is significantly higher than the one observed on the remaining items should consider to create a stratum with these elements.
  - A 100% audit of the high value items should be applied.
  - For the remaining population, determine the size of the sample necessary, using professional judgment and taking account the level of assurance provided by the system audits. A rule of thumb is that the sample size should not be less than 10% of the remaining population of operations, but this value can change according to the auditor professional judgment.
- Option 2: If there are not any high-value operations in the population (with expenditure above the recommended cut-off) the high value stratum cannot be identified. In this design:
  - Compute the size of the sample necessary, based on professional judgement and taking account of the level of assurance provided by the system audits. Again, a rule of thumb is that the sample size should not be less than 10% of the population of operations, but the auditor may revise this threshold using professional judgment.

In any case it is recommended that sample is selected using a random method. In particular the selection can be made either using equal probability selection as done in simple random sampling (cf. sections 7.1.1.6 or 7.1.2.6) or difference estimation (cf. sections 7.2.1.6 or 7.2.2.6) or probability proportional to size (expenditure) selection (as in section 7.4.2 below). The choice should take into account the variability of expenditure in the low-value stratum and the expectation regarding the association between errors and expenditure. Whenever happens to still have significant variability of expenditure in the low-value stratum items and there is an expectation of high positive association between errors and expenditure the probability proportional to size selection should be implemented. Otherwise, the choice should be made over equal probability selection. Non-random selection methods are also a possibility, mainly when the population and the sample sizes are very small. For instance if one has a population of 15 units and plans to select a sample of 2, it may be admissible to select a sample on the basis of the auditor's sound judgment if he/she has information about the characteristics and risks associated to the 15 units in the population.

For the exhaustive stratum, that is, for the stratum containing the sampling units with book value larger cut-off value, the projected error is as follows:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

where $E_i = BV_i - CBV_i, i = 1,2, \dots, n_e$, is as usual the amount of error of an operation, i.e. the difference between the book value of the i-th operation included in sample and the respective corrected book value.

In other words the projected error is the summation of the individual errors for all items in the high-value stratum.

For the stratum containing the sampling units with book value smaller than the cut-off value, the projected error is different whether equal selection probabilities or probability proportional to size was implemented.

If units were selected with equal probabilities, the projected error for the low-value stratum is

$$EE_s = N_s \frac{\sum_{i=1}^{n_s} E_i}{n_s}.$$

where N_s is the population size and n_s the sample size in the low value stratum.

This projected error is equal to the average of errors in the sample multiplied by the population size of the low-value stratum.

If units were selected with probabilities proportional to the value of expenditure, the projected error for the low-value stratum is

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

where BV_s is the total book value and n_s the sample size in the low value stratum.

This projected error is equal to the sample average of error rates multiplied by the total book value of the low-value stratum.

The total projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

The projected error is finally compared to the maximum tolerable error (materiality times the population expenditure):

- If below the tolerable error, then we conclude that the population does not contain material error;
- If above the tolerable error, then we conclude that the population contains material error.

Despite the constraints (i.e. it is not possible to calculate the upper limit of error and consequently there is no control of the audit risk), the projected error rate is the best estimation of the error in the population and can thus be compared with the materiality threshold in order to conclude that the population is (or not) materially misstated.

7.4.2 Example

Let's assume a population of 36 operations for which expenditure 22,031,228 € has been declared.

This population tends to have an insufficient size to be audited through statistical sampling. Further, sampling of payment claims is not possible. Therefore the AA decides to use a non-statistical approach with stratification of the high-value operations since there are a few operations with extremely large expenditure.

The characteristics of the population are summarized below:

Declared expenditure (DE) in the reference period	22,031,228 €
Size of population (operations)	36
Materiality level (maximum 2%)	2%
Tolerable misstatement (TE)	440,625 €

At the first step the auditors will identify the operations which, individually, represent a significant amount or are significant because of their nature. The individually significant amounts are determined as equal to materiality (2% of 22,031,228 €).

The following table summarizes the results:

Number of units above cut-off value	4
Population book value above cut-off	8,411,965 €
Remaining population value	13,619,623 €

These projects will be excluded from sampling and will be treated separately. The total value of these projects is 8,411,965 €. The amount of error found in these four operation amounts to

$$EE_e = 80{,}028.$$

The AA considers that the management and control system has average quality, so it decides to select a sample size of 20% of the remaining population of operations. That is, 20% x 32=6.4 rounded by excess to 7.

Due to the large variability in the expenditure for this population, the auditor decides to select the sample in the remaining population using probability proportional to size (MUS). The sampling interval is equal to the total expenditure in the non-exhaustive stratum (BV_s) divided by the sample size (n_s), i.e.

$$SI = \frac{BV_s}{n_s} = \frac{13{,}619{,}623}{7} = 1{,}945{,}609$$

A file containing the remaining 32 operations of the population is randomly sorted and a sequential cumulative book value variable is created. The sample is selected, selecting each item containing the 1,945,609th monetary unit.

The value of extrapolated error for the low-value stratum is

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_{si}}{BV_{si}}$$

where BV_s is the total book value of the remaining population and n_s the correspondent sample size. Notice that this projected error is equal to the sum of the error rates multiplied by the sampling interval. The sum of the error rates is equal to 0.0272:

$$EE_s = \frac{13{,}619{,}623}{7} \times 0.0272 = 52{,}921.$$

The total extrapolated error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s = 80{,}028 + 52{,}921 = 132{,}949$$

The projected error is finally compared to the maximum tolerable error (2% of 22,031,228 €=440,625 €). The projected error is smaller than the materiality level.

The conclusion that can be derived from the exercise is that the auditor can reasonably conclude that the population does not contain a material error. Nevertheless, the achieved precision cannot be determined and the confidence of the conclusion is unknown. The auditor will therefore have to use his professional judgement to decide whether to apply additional audit procedures or alternative strategies to evaluate the declared expenditure.

8 Selected topics

8.1 How to determine the anticipated error

The anticipated error can be defined as the amount of error the auditor expects to find in the population. Factors relevant to the auditor's consideration of the expected error include the results of the test of controls, the results of audit procedures applied in the prior period and the results of other substantive procedures. One should consider that the more the anticipated error differs from the true error, the higher the risk of reaching inconclusive results after performing the audit (MLE <2% and ULE > 2%).

To set the value of the anticipated error the auditor should take into consideration:

1. If the auditor has information on the error rates of previous years, the anticipated error should, in principal, be based on the projected error obtained in the previous year; nevertheless if the auditor has received information about changes in the quality of the control systems, this information can be used either to reduce or increase the anticipated error. For example, if last year projected error rate was 0.7% and no further information exists, this value can be imputed to the anticipated error rate. If, however the auditor has received evidence about an improvement of the systems that reasonably has convinced him/her that the error rate in the current year will be lower, this information can be used to reduce the anticipated error to a smaller value of, for example, 0,4%.
2. If there is no historical information about error rates, the auditor can use a preliminary/pilot sample in order to obtain an initial estimate of the population error rate. The anticipated error rate is considered to be equal to the projected error from this preliminary sample. If a preliminary sample is already being selected, in order to compute the standard-deviations necessary to calculate the formulas for sample size, then this same preliminary sample can also be used to compute an initial projection of the error rate and thus of the anticipated error.
3. If there is no historical information to produce an anticipated error and a preliminary sample cannot be used due to uncontrollable restrictions, then the auditor should set a value to the anticipated error based on professional experience and judgment. In this situation, it is usual to choose an anticipated

error between 10% to 30% of the materiality. Nevertheless, the value should not be limited by this guideline and should mostly reflect the auditor expectation regarding the true level of error in the population.

In summary, the auditor should use historical data, auxiliary data, professional judgement or a mix of the above to choose a value as realistic as possible for the anticipated error.

An anticipated error based on objective quantitative data is usually more accurate and avoids carrying out additional work in the case audit results are inconclusive. For example if the auditor sets an anticipated error of 10% of materiality, i.e. 0,2% of expenditure, and at the end of the audit he obtains a projected error of 1,5%, results will most probably be inconclusive as the upper limit of error will be higher than the materiality level, and additional work will be required. To avoid these situations the auditor should use as anticipated error, in future sampling exercises, the most realistic possible measure of the true error in the population.

A special situation may arise when the anticipated error rate is in the neighbourhood of 2%. For example, if the anticipated error is 1,9% and the confidence level is high (e.g. 90%) it may happen that the resulting sample size is extremely large and hardly achievable. This phenomenon is common to all sampling methods and happens when the planned precision is very small (0,1% in the example). An advisable possibility, under this situation, is to divide the population in two different subpopulations where the auditor expects to find different levels of error. If it is possible to identify one subpopulation with expected error below 2% and other subpopulation for which the expected error is above 2%, the auditor can safely plan two different samples for these subpopulations, without the risk of obtaining too large samples sizes.

Finally, the Audit Authority should plan its audit work in a way to achieve sufficient precision of the MLE even when the anticipated error is well above materiality (i.e. equal or above 3,5%). In this case it is advisable to compute the sample size formulas with an anticipated error resulting in a maximum planned precision of 1,5%. In other cases where historical data on audits of operations and possibly system audit results lead to a very low anticipated error rate (e.g. not higher than 0,5%, thus leading to a planned precision above 1,5%), the auditor may decide to use this historical data or 0,5% as anticipated error. The audit authorities should be aware that the goal of this approach is to take adequate measures to ensure that the achieved precision, after carrying out audit work, would not be above 2% in which case the calculated MLE may not be sufficiently precise.

8.2 Additional sampling

8.2.1 Complementary sampling (due to insufficient coverage of high risk areas)

In Article 17 § 5 of the Commission Regulation (EC) No 1828/2006 and Article 43 § 5 of the Commission Regulation (EC) No 498/2007, reference is made to complementary sampling.

The results of the random statistical sampling have to be assessed in relation to the results of the risk analysis of each programme and to the coverage of priorities, type of operations, beneficiaries etc. in the programme. Where it is concluded from this comparison that the random statistical sample does not address the high risk areas and/or coverage, it should be completed by a further selection of operations, i.e. a complementary sample.

The audit authority should make this assessment on a regular basis during the implementation period.

In this framework, the results of the audits covering the complementary sample are analysed separately from the results of the audits covering the random statistical sample. In particular, the errors detected in the complementary sample are not taken into account for the calculation of the error rate resulting from the audit of the random statistical sample. However, a detailed analysis must also be done of the errors identified in the complementary sample, in order to identify the nature of the errors and to provide recommendations to correct them.

The results of the complementary sample should be reported to the Commission in the Annual Control report immediately following the audit of a complementary sample.

8.2.2 Additional sampling (due to inconclusive results of the audit)

Whenever the results of the audit are inconclusive and, after considering section 8.7, additional work is needed (typically when the projected error is below the materiality but the upper limit is above) an option is to select an additional sample. For this, the projected error produced from the original sample should be substituted in formulas for sample size determination in the place of the anticipated error (in fact the projected error is at that moment the best estimate of the error in the population). Doing this, a new sample size can be calculated based on the new information arising from the original sample. The size of the additional sample needed can be obtained by subtracting the original sample size from the new sample size. Finally a new sample can be selected (using the same method as for the original sample), the two samples are grouped together and results (projected error and precision) should be recalculated using data from the final grouped sample.

Imagine that the original sample with sample size equal to 60 operations produced a projected error rate of 1.5%, with a precision of 0.9%. Consequently the upper limit for the error rate is 1.5+0.9=2.4%. In this situation we have a projected error rate that is below the 2% materiality level, but an upper limit that it is above. Consequently, the auditor faces a situation where further work is needed to achieve a conclusion (cf. Section 5.11). Among the alternatives one can choose to carry out further testing through additional sampling. If this is the choice, the projected error rate of 1.5% should be imputed in the formula for sample size determination in the place of the anticipated error, leading to a recalculation of the sample size, which would produce in our example a new sample size of n=78. As the original sample had a size of 60 operations, this value should be subtracted from the new sample size resulting in 78-60=18 new observations. Therefore an additional sample of 18 operations should be now selected from the population using the same method as for the original sample (ex. MUS). After this selection, the two samples are grouped together forming a new whole sample of 60+18=78 operations. This global sample will finally be used to recalculate the projected error and the precision of the projection using the usual formulas.

8.3 Sampling carried out during the year

The audit authority may decide to carry out the sampling process in several periods during the year (typically two semesters). A first audit will cover the operations and expenditure referring to the period 01/01/xx-30-06-xx and a second audit will cover the population and expenditure of the following semester 01/07/xx-31-12-xx. This approach should not be used with the goal of reducing the global sample size. In general the sum of sample sizes for the several periods of observation will be larger than the sample size that would be obtained by carrying out sampling in one single period at the end of the year. Nevertheless, if calculations are based on realistic assumptions, usually the sum of the partial sample sizes would not be dramatically larger than the one produced in a single observation. The major advantage of this approach is not related with sample size reduction, but mainly allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

This approach requires that at the first observation period some assumptions are made in regard to the subsequent observation periods (typically the next semester). For example, the auditor may need to produce an estimate of the total expenditure expected to be found in the population in the next semester. This means that this method is not implemented without risk, due to possible inaccuracies in the assumptions related to following periods. If characteristics of the population in the following periods differ significantly from the assumptions, sample size for the following period may have to be increased and the global sample size (including all periods) may be larger than the one expected and planned.

Section 7 of this guidance presents the specific formulas and detailed guidance for implementing sampling in two observation periods within one year. Note that this approach can be followed with any sampling method that has been chosen by the auditor, including possible stratification. It is also acceptable to treat the several periods of the year as different populations from which different samples are planned and extracted[23]. This is not dealt with in the methods proposed in Section 7 as its application is straightforward using the standard formulas for the several sampling methods. Under this approach the only additional work is to add together the partial projected errors at the end of the year.

The audit authority should aim at using the same sampling method for a given reference year. The use of different sampling methods in the same reference year is not encouraged, as this would result in more complex formulas to extrapolate the error for that year. Namely, global precision measures can be produced, provided that statistical sampling was implemented in the same reference year. However, these more complex formulas are not included in the present document. Hence, if the audit authority uses different sampling methods in the same year, it should seek the adequate expertise in order to obtain the correct calculation of the projected error rate.

8.4 Change of sampling method during the programming period

If the audit authority is of the opinion that the sampling method initially selected is not the most appropriate one, it could decide to change the method. However, this should be notified to the Commission in the framework of the Annual Control Report or in a revised audit strategy.

8.5 Error rates

Formulas and methodology presented in section 7 to produce projected error and the respective precision are thought for errors in terms of monetary units, i.e. the difference between the book value in the population (declared expenditure) and the correct/audited book value. Nevertheless, it is common practice to produce results in the form of error rates as they are appealing due to their intuitive interpretation. The conversion of errors into error rates is straightforward and common to all sampling methods.

The projected error rate is simply equal to the projected error divided by the book value in the population

[23] This will of course result in sample sizes larger than the ones offered by the approach presented in Section 7.

$$EER = \frac{EE}{BV}$$

Similarly, the precision for the estimation of the error rate is equal to the precision of the projected error divided by the book value

$$SER = \frac{SE}{BV}$$

8.6 Two-stage sampling

In general, all the expenditure declared to the Commission for all the selected operations in the sample should be subject to audit. Nevertheless, whenever the selected operations include a large number of payment claims or invoices they can be audited through sampling, selecting the claims/invoices by using the same principles used to select the operations. This situation corresponds to what is known as two-stage sampling. Whenever this approach is followed, the sampling methodology should be recorded in the audit report or working papers.

In this case, appropriate sample sizes should be calculated within each operation. The exact determination of sample size for two stage sampling is out of the scope of these notes, but are in everything similar to the strategy used when selecting the operations at the first stage of sampling .In other words, you can use the same sample size determination formulas to calculate the sample size of invoices or payment claims needed to project the operation expenditure with a certain planned precision (for ex. 2% of its expenditure). Despite the sampling methodology used to determine sample sizes, a basic rule of thumb is to never use sample sizes smaller than 30 observations (i.e., invoices or payment claims from beneficiaries).

The easiest way to select the sample of the second stage (claims or invoices) is using simple random sampling[24] (see Section 7.1.1 for the formula to determine the sample size). Whenever you select this sample with equal probabilities, the projection procedure is very straightforward as you can use the sample error rate as an estimator of the operation error rate. Once you have estimated the error rates for every operation in the sample, the projection for the population follows the usual procedure (as if you had observed the whole expenditure of the operation).

For example if an operation in the sample has an expenditure of 2,500,000€ and 400 invoices. If you decide to select a sample of 40 invoices, you should first calculate the total error and the total expenditure in the sample. Imagine that the total expenditure is

[24] The audit authority may choose to use more sophisticated methods for selecting the claims/invoices within the operations. A stratification of claims/invoices by level of expenditure or a selection based on probabilities proportional to size (as in MUS) are some possible examples.

290,000€ and the total error 9,280€. The estimated error rate for the operation is 3,2%=(9,280€/290,000€) and the projected error of the operation is 80,000€=3,2%*2,500,000€.

8.7 Recalculation of the confidence level

When after performing the audit, the AA finds that the projected error is lower than the materiality level but the upper limit is larger than that threshold, it may want to recalculate the confidence level that would generate conclusive results (i.e. to have both the projected error and the upper limit below materiality).

When this recalculated confidence level is still compatible with an assessment of the quality of the management and control systems (see table in section 3.2), it will be perfectly safe to conclude that the population is not materially misstated even without carrying out additional audit work. Therefore, only in situations where the recalculated confidence is not acceptable (not in accordance with the assessment of the systems) is necessary to proceed with the additional work suggested in Section 5.11.

The recalculation of the confidence interval is performed as follows:

- Calculate the materiality level in value, i.e. the materiality level (2%) times the total book value of the population.

- Subtract the projected error (EE) from the materiality value.

- Divide this result by the precision of the projection (SE). This precision is dependent on the sampling method and presented in the sections devoted to the presentation of the methods.

- Multiply the above result by the z parameter used both for sample size and precision calculation and obtain a new value z^*

$$z^* = z \times \frac{(0.02 \times BV) - EE}{SE}$$

- Look for the confidence level associated to this new parameter (z^*) in a table of the normal distribution (in appendix). Alternatively you can use the following excel formula "=1-(1-NORMSDIST(z^*))*2".

Example: after auditing a population with a book value of 1,858,233,036€ and a confidence level of 90% (corresponding to $z = 1.645$, cf. Section 6.4), we have obtained the following results

Characteristic	Value
BV	1,858,233,036€
Materiality (2% of BV)	37,164,661€
Projected error (EE)	14,568,765€ (0.8%)
Precision (SE)	26,195,819€ (1.4%)
Upper error limit (ULE)	40,764,584€ (2.2%)

The new z^* parameter is obtained as

$$z^* = 1.645 \times \frac{37{,}164{,}661€ - 14{,}568{,}765€}{26{,}195{,}819€} = 1.419$$

Using the MS Excel function "=1-(1-NORMSDIST(1.419))*2", we obtain the new confidence level 84,4%.

Being this recalculated confidence level compatible with the assessment about the quality of the management and control systems one can conclude that the population is not materially misstated.

8.8 Sampling technique applicable to system audits

8.8.1 Introduction

Article 62 of Council Regulations (EC) № 1083/2006 states: "The audit authority of an operational programme shall be responsible in particular for: (a) ensuring that audits are carried out to verify the effective functioning of the management and control system of an operational programme…". These audits are called system audits. System audits aim at testing the effectiveness of controls in the management and control system and concluding on the assurance level that can be obtained from the system. Whether or not to use a statistical sampling approach for the test of controls is a matter of professional judgement regarding the most efficient manner to obtain sufficient appropriate audit evidence in the particular circumstances.

Since for system audits the auditor's analysis of the nature and cause of errors is important, as well as, the mere absence or presence of errors, a non-statistical approach could be appropriate. The auditor can in this case choose a fixed sample size of the items to be tested for each key control. Nonetheless, professional judgment will have to be used in applying the relevant factors[25] to consider. If a non-statistical approach is used then the results cannot be extrapolated.

Attribute sampling is a statistical approach which can help the auditor to determine the level of assurance of the system and to assess the rate at which errors appear in a

[25] For further explanation or examples see "Audit Guide on Sampling, American Institute of Certified Public Accountants, 01/04/2001".

sample. Its most common use in auditing is to test the rate of deviation from a prescribed control to support the auditor's assessed level of control risk. The results can then be projected to the population.

As a generic method encompassing several variants, attribute sampling is the basic statistical method to apply in the case of system audits; any other method that can be applied to system audits will be based on the concepts developed below.

Attribute sampling tackles binary problems such as yes or no, high or low, true or false answers. Through this method, the information relating to the sample is projected to the population in order to determine whether the population belongs to one category or the other.

The Regulation does not make it obligatory to apply a statistical approach to sampling for control tests in the scope of a systems audit. Therefore, this chapter and the related annexes are included for general information and will not be developed further.

For further information and examples related to the sampling techniques applicable to system audits, please refer to the specialized audit sampling literature.

When applying attribute sampling in a system audit, the following generic six-step plan should be applied.

1. Define the test objectives: for instance, determine whether the error frequency in a population meets the criteria for a high assurance level;
2. Define the population and sampling unit: for instance the invoices allocated to a programme;
3. Define the deviation condition: this is the attribute being assessed, e.g. the presence of a signature on the invoices allocated to an operation within a programme;
4. Determine the sample size, according to the formula below;
5. Select the sample and carry out the audit (the sample should be selected randomly);
6. Evaluate and document the results.

8.8.2 Sample size

Computing sample size n within the framework of attribute sampling relies on the following information:

- Confidence level and the related coefficient z from a normal distribution (see Section 6.4)

- Maximum tolerable deviation rate, **T**, determined by the auditor; the tolerable levels are set by the Member State audit authority (e.g. the number of missing signatures on invoices under which the auditor considers there is no issue);
- The anticipated population deviation rate, p, estimated or observed from a preliminary sample. Note that the tolerable deviation rate should be higher than the expected population deviation rate, as, if that is not the case, the test has no purpose (i.e. if you expect an error rate of 10%, setting a tolerable error rate of 5% is pointless because you expect to find more errors in the population than you are willing to tolerate).

The sample size is computed as follows[26]:

$$n = \frac{z^2 \times p \times (1-p)}{T^2}.$$

Example: assuming a confidence level of 95% ($z = 1.96$), a tolerable deviation rate (T) of 12% and an expected population deviation rate (p) of 6%, the minimum sample size would be

$$n = \frac{1.96^2 \times 0.06 \times (1-0.06)}{0.12^2} \approx 16.$$

Note that the population size has no impact on the sample size; the calculation above slightly overstates the required sample size for small populations, which is accepted. Ways to reduce the required sample size include reducing the confidence level (i.e. raising the risk of assessing the control risk too low) and raising the tolerable deviation rate.

8.8.3 Extrapolation

The number of deviations observed in the sample divided by the number of items in the sample (i.e. the sample size) is the sample deviation rate:

$$EDR = \frac{\#\, of\ deviations\ in\ the\ sample}{n}$$

This is also the best estimator of the extrapolated deviation rate (EDR) one can obtain from the sample.

[26] When dealing with a small population size, i.e. if the final sample size represents a large proportion of the population (as a rule of thumb more than 10% of the population) a more exact formula can be used leading to $n = \frac{z^2 \times p \times (1-p)}{T^2} \Big/ \left(1 + \frac{z^2 \times p \times (1-p)}{T^2}\right)$.

8.8.4 Precision

Remember that precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). The precision is given by the following formula

$$SE = z \times \frac{p_s \times (1 - p_s)}{\sqrt{n}}$$

where p_s is the ratio of number of deviations observed in the sample to the sample size, the sample deviation rate.

8.8.5 Evaluation

The achieved upper deviation limit is a theoretical figure based on the sample size and the number of errors encountered:

$$ULD = EDR + SE.$$

It represents the maximum error rate of the population at the defined confidence level and results from binomial tables (for instance, for sample size 150 and an observed amount of deviations of 3 (sample deviation rate of 2%), the maximum deviation rate (or achieved upper deviation limit) at a 95% confidence level is:

$$ULD = \frac{3}{150} + 1.96 \times \frac{\frac{3}{150} \times \left(1 - \frac{3}{150}\right)}{\sqrt{150}} = 0.023.$$

If this percentage is higher than the tolerable deviation rate, the sample does not support the assumed expected error rate of the population at that confidence level. The logical conclusion is therefore that the population does not meet the criterion set of high assurance level and must be classified as having an average or low assurance level. Note that the threshold at which low, average or high assurance is reached is defined by the AA.

8.8.6 Specialised methods of attribute sampling

Attribute sampling is a generic method, and therefore some variants have been designed for specific purposes. Among those, discovery sampling and stop-or-go sampling serve specialised needs.

<u>Discovery sampling</u> aims at auditing cases where a single error would be critical; it is therefore particularly geared towards the detection of cases of fraud or avoidance of

controls. Based on attribute sampling, this method assumes a zero (or at least very low) rate of error and is not well suited for projecting the results to the population, should errors be found in the sample. Discovery sampling allows the auditor to conclude, based on a sample, whether the assumed very low or zero error rate in the population is a valid assumption. It is not a valid method for assessing the level of assurance of internal controls, and therefore is not applicable to system audits.

Stop-or-go sampling comes out of the frequent need to reduce the sample size as much as possible. This method aims at concluding that the error rate of the population is below a predefined level at a given confidence level by examining as few sample items as possible – the sampling stops as soon as the expected result is reached. This method is also not well-suited for projecting the results to the population, though it can be useful for assessing system audit conclusions. It can be used when the outcome of system audits is questioned, to check whether the criterion is indeed reached for the assurance level provided.

Appendix 1 – Projection of random errors when systemic errors are identified

1. Introduction

The purpose of this appendix is to clarify the calculation of the projected random errors when systemic errors are identified. As explained in the guidance (cf. section 4.1), the identification of a potential systemic error implies carrying out the complementary work necessary for the identification of its total extent and subsequent quantification. This means that all the situations susceptible of containing an error of the same type as the one detected in the sample should be identified, thus allowing the delimitation of its total effect in the population. If such delimitation is not done before the ACR is submitted, the systemic errors are to be treated as random for the purposes of the calculation of the projected random error.

The total projected error (TPE) corresponds to the sum of the following errors: projected random errors, systemic errors and uncorrected anomalous errors.

In this context, when extrapolating the random errors found in the sample to the population, the Audit Authority should deduct the amount of systemic error from the book value (total expenditure certified in the reference year) whenever this value is part of the projection formula, as explained below.

As regard mean-per-unit estimation[27] and difference estimation, there is no change in the formulas presented in the guidance for the projection of random errors. For monetary unit sampling this appendix sets out two possible approaches (one approach that does not change the formula and another approach that requires formulas that are more complex in order to obtain better precision). For ratio estimation, the projection of the random errors and the calculation of the precision (SE) requires the use of the total book value deducted from systemic errors.

In all statistical sampling methods, when systemic errors or anomalous non-corrected errors exist, the upper limit of error (ULE) corresponds to the sum of the TPE plus the precision (SE). When only random errors exist, the ULE is the sum of the projected random errors plus the precision.

In the following sections a more detailed explanation about the extrapolation of random errors in the presence of systemic errors for the most important sampling techniques is offered.

[27] cf. section on "simple random sampling" in the guidance.

2. Simple random sampling

2.2 Mean-per-unit estimation

The projection of random errors and the calculation of precision are as usual:

$$EE_1 = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$

$$SE_1 = N \times z \times \frac{s_e}{\sqrt{n}}$$

where E_i represents the amount of random error found in each sampling unit and s_e is, as usual, the standard-deviation of random errors in the sample.

The total projected error is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

The upper limit of error (ULE) is equal to the summation of the total projected error, TPE, and the precision of the extrapolation

$$ULE = TPE + SE$$

2.3 Ratio estimation

The projection of the random error is:

$$EE_2 = BV' \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV'_i}$$

where BV' represents the total book value of the population deducted from systemic errors that were previously delimited, $BV' = BV - systemic\ errors$. BV'_i is the book value of unit *i* deducted by the amount of systemic error affecting that unit.

The sample error rate in the above formula is just the division of the total amount of random error in the sample by the total amount of expenditure (deducted from systemic errors) of units in the sample (expenditure audited).

The precision is given by the formula

$$SE_2 = N \times z \times \frac{s_{q'}}{\sqrt{n}}$$

where $s_{q'}$ is the sample standard deviation of the variable q':

$$q'_i = E_i - \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV'_i} \times BV'_i.$$

This variable is for each unit in the sample computed as the difference between its random error and the product between its book value (deducted from systemic errors) and the error rate in the sample.

The total projected error is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

The upper limit of error (ULE) is equal to the summation of the total projected error, TPE, and the precision of the extrapolation

$$ULE = TPE + SE$$

3. Difference estimation

The projected random error at the level of the population can be computed as usual by multiplying the average random error observed per operation in the sample by the number of operations in the population, yielding the projected error

$$EE = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$ [28]

In a second step the total projected error, TPE, should be computed adding the amount of systemic error and anomalous non corrected errors to the random projected error (EE).

The correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected subtracting the total projected error (TPE) from the book value (BV) in the population (declared expenditure without deducting the systemic errors). The projection for the correct book value (CBV) is

$$CBV = BV - TPE$$

[28] Alternatively the projected random error can be obtained using the formula proposed under ratio estimation $EE_2 = BV' \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV'_i}$.

The precision of the projection is, as usual, given by

$$SE = N \times z \times \frac{s_e}{\sqrt{n}}$$

where s_e is the standard-deviation of random errors in the sample.

To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is, as usual, equal to

$$LL = CBV - SE$$

The projection for the correct book value and the upper limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE), which corresponds to the materiality level times the book value:

$$BV - TE = BV - 2\% \times BV = 98\% \times BV$$

The evaluation of the error should be done in accordance with section 7.2.1.5 of the guidance.

4. Monetary unit sampling-standard approach

There are two possible approaches to project random errors and calculate precision under monetary unit sampling in the presence of systemic errors. They will be referred as *MUS standard approach* and *MUS ratio estimation*. The second method is based on a more complex calculation. Although, they can both be used in any scenario, the second method will generally produce more precise results when the random errors are more correlated with the book values corrected from the systemic error than with the original book values. When the level of systemic errors in the population is small, the precision gain originated by the second method will usually be very modest and the first method may be a preferable choice due to its simplicity of application.

4.1 MUS standard approach

The projection random errors and the calculation of precision are performed as usual.

The projection of the random errors to the population should be made differently for the units in the exhaustive stratum and for the items in the non-exhaustive stratum.

For the <u>exhaustive stratum</u>, that is, for the stratum containing the sampling items with book value larger than the cut-off ($BV_i > \frac{BV}{n}$) the projected error is just the summation of the errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the <u>non-exhaustive stratum</u>, i.e. the stratum containing the sampling items with book value smaller or equal to the cut-off value ($BV_i \leq \frac{BV}{n}$) the projected random error is

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

Note that the book values mentioned in the above formula refer to the expenditure **without** subtracting the amount of systemic error. This means that the error rates, $\frac{E_i}{BV_i}$, should be calculated using the total expenditure of the sample units despite a systemic error was or not found in each unit.

The precision is also given by the usual formula:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r$$

where s_r is the standard-deviation of random error rates in the sample of the non-exhaustive stratum. Again this error rates should be calculated using the original book values, BV_i, **without** subtracting the amount of systemic error.

The total projected error is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

The upper limit of error (ULE) is equal to the summation of the total projected error, TPE, and the precision of the extrapolation

$$ULE = TPE + SE$$

4.2 MUS ratio estimation

The projection of the random errors to the population should again be made differently for the items in the exhaustive stratum and for the items in the non-exhaustive stratum.

For the exhaustive stratum, that is, for the stratum containing the sampling units with book value larger than the cut-off ($BV_i > \frac{BV}{n}$) the projected error is just the summation of the random errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the non-exhaustive stratum, i.e. the stratum containing the sampling units with book value smaller or equal to the cut-off value ($BV_i \leq \frac{BV}{n}$) the projected random error is

$$EE_s = BV'_s \times \frac{\sum_{i=1}^{n_s} \frac{E_i}{BV_i}}{\sum_{i=1}^{n_s} \frac{BV'_i}{BV_i}}$$

where BV'_s represents the total book value of the low-value stratum deducted from systemic errors that were previously delimited in the same stratum, $BV'_s = BV_s - systemic\ errors\ in\ the\ sampling\ stratum$. BV'_i is the book value of unit *i* deducted by the amount of systemic error affecting that unit.

The precision is given by the formula:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_{rq}$$

where s_{rq} is the standard-deviation of the error rates for the **transformed error** q'. To calculate this formula, it is first necessary to calculate the values of the **transformed errors** for all units in the sample:

$$q'_i = E_i - \frac{\sum_{i=1}^{n_s} \frac{E_i}{BV_i}}{\sum_{i=1}^{n_s} \frac{BV'_i}{BV_i}} \times BV'_i.$$

Finally, the standard-deviation of error rates in the sample of the non-exhaustive stratum (s_{rq}), for the transformed error q', is obtained as:

$$s_{rq} = \sqrt{\frac{1}{n_s - 1}\sum_{i=1}^{n_s}\left(\frac{q'_i}{BV_{i_i}} - \overline{rq}_s\right)^2}$$

having $\overline{rq}_s$ equal to the simple average of the transformed error rates in the sample of the stratum

$$\bar{r}q_s = \frac{\sum_{i=1}^{n_s}\frac{q'_i}{BV_i}}{n_s}$$

The total projected error is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

The upper limit of error (ULE) is equal to the summation of the total projected error (TPE), and the precision of the extrapolation

$$ULE = TPE + SE$$

5. Non-statistical sampling

The projection and the calculation of precision are performed as usual.

If an exhaustive stratum exists, that is, a stratum containing the sampling units with book value larger cut-off value, the projected error is just the sum of random errors found in this group:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the sampling stratum, if units were selected with equal probabilities, the projected random error is as usual

$$EE_s = N_s \frac{\sum_{i=1}^{n_s} E_i}{n_s}.$$

where N_s is the population size and n_s the sample size in the low value stratum

If units were selected with probabilities proportional to the value of expenditure, the projected random error for the low-value stratum is

$$EE_s = \frac{BV_s}{n_s}\sum_{i=1}^{n_s}\frac{E_i}{BV_i}$$

where BV_s is the total book value (**without** deducting the amount of systemic error), BV_i the book value of sample unit i (**without** deducting the amount of systemic error and n_s the sample size in the low value stratum.

Similarly to what has been presented for MUS method (cf. Section 2.4) the ratio estimation formula,

$$EE_s = BV'_s \times \frac{\sum_{i=1}^{n_s}\frac{E_i}{BV_i}}{\sum_{i=1}^{n_s}\frac{BV'_i}{BV_i}}$$

can alternatively be used. Again BV'_s represents the total book value of the low-value stratum deducted from systemic errors that were previously delimited in the same stratum, $BV'_s = BV_s - systemic\ errors\ in\ the\ sampling\ stratum.$ BV'_i is the book value of unit i deducted by the amount of systemic error affecting that unit.

The total projected error (TPE) is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

Appendix 2 – Reliability factors for MUS

	Risk of incorrect acceptation									
Number of errors	1%	5%	10%	15%	20%	25%	30%	37%	40%	50%
0	4.61	3.00	2.30	1.90	1.61	1.39	1.20	0.99	0.92	0.69
1	6.64	4.74	3.89	3.37	2.99	2.69	2.44	2.14	2.02	1.68
2	8.41	6.30	5.32	4.72	4.28	3.92	3.62	3.25	3.11	2.67
3	10.05	7.75	6.68	6.01	5.52	5.11	4.76	4.34	4.18	3.67
4	11.60	9.15	7.99	7.27	6.72	6.27	5.89	5.42	5.24	4.67
5	13.11	10.51	9.27	8.49	7.91	7.42	7.01	6.49	6.29	5.67
6	14.57	11.84	10.53	9.70	9.08	8.56	8.11	7.56	7.34	6.67
7	16.00	13.15	11.77	10.90	10.23	9.68	9.21	8.62	8.39	7.67
8	17.40	14.43	12.99	12.08	11.38	10.80	10.30	9.68	9.43	8.67
9	18.78	15.71	14.21	13.25	12.52	11.91	11.39	10.73	10.48	9.67
10	20.14	16.96	15.41	14.41	13.65	13.02	12.47	11.79	11.52	10.67
11	21.49	18.21	16.60	15.57	14.78	14.12	13.55	12.84	12.55	11.67
12	22.82	19.44	17.78	16.71	15.90	15.22	14.62	13.88	13.59	12.67
13	24.14	20.67	18.96	17.86	17.01	16.31	15.70	14.93	14.62	13.67
14	25.45	21.89	20.13	19.00	18.13	17.40	16.77	15.97	15.66	14.67
15	26.74	23.10	21.29	20.13	19.23	18.49	17.83	17.02	16.69	15.67
16	28.03	24.30	22.45	21.26	20.34	19.57	18.90	18.06	17.72	16.67
17	29.31	25.50	23.61	22.38	21.44	20.65	19.96	19.10	18.75	17.67
18	30.58	26.69	24.76	23.50	22.54	21.73	21.02	20.14	19.78	18.67
19	31.85	27.88	25.90	24.62	23.63	22.81	22.08	21.17	20.81	19.67
20	33.10	29.06	27.05	25.74	24.73	23.88	23.14	22.21	21.84	20.67
21	34.35	30.24	28.18	26.85	25.82	24.96	24.20	23.25	22.87	21.67
22	35.60	31.41	29.32	27.96	26.91	26.03	25.25	24.28	23.89	22.67
23	36.84	32.59	30.45	29.07	28.00	27.10	26.31	25.32	24.92	23.67
24	38.08	33.75	31.58	30.17	29.08	28.17	27.36	26.35	25.95	24.67
25	39.31	34.92	32.71	31.28	30.17	29.23	28.41	27.38	26.97	25.67
26	40.53	36.08	33.84	32.38	31.25	30.30	29.46	28.42	28.00	26.67
27	41.76	37.23	34.96	33.48	32.33	31.36	30.52	29.45	29.02	27.67
28	42.98	38.39	36.08	34.57	33.41	32.43	31.56	30.48	30.04	28.67
29	44.19	39.54	37.20	35.67	34.49	33.49	32.61	31.51	31.07	29.67
30	45.40	40.69	38.32	36.76	35.56	34.55	33.66	32.54	32.09	30.67
31	46.61	41.84	39.43	37.86	36.64	35.61	34.71	33.57	33.11	31.67
32	47.81	42.98	40.54	38.95	37.71	36.67	35.75	34.60	34.14	32.67
33	49.01	44.13	41.65	40.04	38.79	37.73	36.80	35.63	35.16	33.67
34	50.21	45.27	42.76	41.13	39.86	38.79	37.84	36.66	36.18	34.67
35	51.41	46.40	43.87	42.22	40.93	39.85	38.89	37.68	37.20	35.67
36	52.60	47.54	44.98	43.30	42.00	40.90	39.93	38.71	38.22	36.67
37	53.79	48.68	46.08	44.39	43.07	41.96	40.98	39.74	39.24	37.67
38	54.98	49.81	47.19	45.47	44.14	43.01	42.02	40.77	40.26	38.67
39	56.16	50.94	48.29	46.55	45.20	44.07	43.06	41.79	41.28	39.67
40	57.35	52.07	49.39	47.63	46.27	45.12	44.10	42.82	42.30	40.67
41	58.53	53.20	50.49	48.72	47.33	46.17	45.14	43.84	43.32	41.67
42	59.71	54.32	51.59	49.80	48.40	47.22	46.18	44.87	44.34	42.67
43	60.88	55.45	52.69	50.87	49.46	48.27	47.22	45.90	45.36	43.67
44	62.06	56.57	53.78	51.95	50.53	49.32	48.26	46.92	46.38	44.67
45	63.23	57.69	54.88	53.03	51.59	50.38	49.30	47.95	47.40	45.67
46	64.40	58.82	55.97	54.11	52.65	51.42	50.34	48.97	48.42	46.67
47	65.57	59.94	57.07	55.18	53.71	52.47	51.38	49.99	49.44	47.67
48	66.74	61.05	58.16	56.26	54.77	53.52	52.42	51.02	50.45	48.67
49	67.90	62.17	59.25	57.33	55.83	54.57	53.45	52.04	51.47	49.67
50	69.07	63.29	60.34	58.40	56.89	55.62	54.49	53.06	52.49	50.67

Appendix 3 – Values for the standardized normal distribution (z)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999064	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999930	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967
4.0	0.999968	0.999970	0.999971	0.999972	0.999973	0.999974	0.999975	0.999976	0.999977	0.999978

Appendix 4 – MS Excel formulas to assist in sampling methods

The formulas listed below can be used in MS Excel to assist in computing the various parameters required by the methods and concepts detailed in this guidance. For further information on the way these formulas work, you can refer to the Excel "help" file that provides the details of the underlying mathematical formulas.

In the above formulas (.) means a vector containing the address of the cells with the values of the sample or population.

=AVERAGE(.) : mean of a data set
=VAR(.) : variance of a sample data set
=VARP(.) : variance of a population data set
=STDEV(.) : standard deviation of a sample data set
=STDEVP(.) : standard deviation of a population data set
=RAND() : random number between 0 and 1, taken from a uniform distribution
=SUM(.) : sum of a data set

Appendix 5 – Glossary

Term	Definition
Anomalous error	An error/misstatement that is demonstrably not representative of the population. A statistical sample is representative for the population and therefore anomalous errors should only be accepted in very exceptional, well-motivated circumstances.
Anticipated error (AE)	The anticipated error is the amount of error the auditor expects to find in the population (after performing the audit). For sample size planning purposes the anticipated error rate is set to a maximum of 4% of the book value of the population.
Attribute sampling	Is a statistical approach to determine the level of assurance of the system and to assess the rate at which errors appear in a sample. Its most common use in auditing is to test the rate of deviation from a prescribed control to support the auditor's assessed level of control risk.
Audit assurance	The assurance model is the opposite of the risk model. If the audit risk is considered to be 5%, the audit assurance is considered to be 95%. The use of the audit assurance model relates to the planning and the underlying resource allocation for a particular programme or group of programmes.
Audit risk *(AR)*	Is the risk that the auditor issues an unqualified opinion, when the declaration of expenditure contains material errors.
Basic precision (*BP*)	Is used in Conservative MUS and corresponds the product between sampling interval and the reliability factor (RF) (already used for calculating sample size).
Book value *(BV)*	The expenditure certified to the Commission of an item (operation/payment claim), $BV_i, i = 1,2, \ldots, N$. The total book value of a population comprises the sum of item book values in the population.
Confidence interval	The interval that contains the true (unknown) population value (in general the amount of error or the error rate) with a certain probability (called confidence level).

Term	Definition
Confidence level	The probability that a confidence interval produced by sample data contains the true population error (unknown).
Control risk *(CR)*	Is the perceived level of risk that a material error in the client's financial statements, or underlying levels of aggregation, will not be prevented, detected and corrected by the management's internal control procedures.
Correct book value *(CBV)*	The correct expenditure that would be found if all the operations/payments claims in the population were audited.
Detection risk	Is the perceived level of risk that a material error in the client's financial statements, or underlying levels of aggregation, will not be detected by the auditor. Detection risks are related to performing audits of operations.
Difference estimation	Is a statistical sampling method based on selection with equal probabilities. The method relies on extrapolating the error in the sample. The extrapolated error is subtracted to the total declared expenditure in the population in order to assess the correct expenditure in the population (i.e. the expenditure that would be obtained if all the operations in the population were audited).
Error *(E)*	For the purposes of this guidance, an error is a quantifiable overstatement of the expenditure certified to the Commission. Is defined as the difference between the book value of the *i*-th item included in sample and the respective correct book value, $\mathrm{E_i} = \mathrm{BV_i} - \mathrm{CBV_i}, \mathrm{i} = 1,2,\ldots,\mathrm{N}$. If the population is stratified, an index h is used to denote the respective stratum: $\mathrm{E_{hi}} = \mathrm{BV_{hi}} - \mathrm{CBV_{hi}}$, where $\mathrm{i} = 1,2,\ldots;\mathrm{N_h}, \mathrm{h} = 1,2,\ldots,\mathrm{H}$ and H is the number of strata.
Expansion factor (*EF*)	Is a factor used in the calculation of conservative MUS when errors are expected, which is based upon the risk of incorrect acceptance. It reduces the sampling error. If no errors are expected, the anticipated error (AE) will be zero and the expansion factor is not used. Values for the expansion factor are found in section 7.3.4.2 of this guidance

Term	Definition
Incremental allowance (*IA*)	The incremental allowance measures the increment in the level of precision introduced by each error found in the sample. This allowance in used in the conservative approach to MUS and should be added to the basic precision value whenever errors are found in the sample (cf. section 7.3.4.5 of this guidance).
Inherent risk *(IR)*	Is the perceived level of risk that a material error may occur in the certified statements of expenditure to the Commission or underlying levels of aggregation, in the absence of internal control procedures. The inherent risk needs to be assessed before starting detailed audit procedures through interviews with management and key personnel, reviewing contextual information such as organisation charts, manuals and internal/external documents.
Irregularity	Same meaning as error.
Known error	An error found in the sample can lead the auditor to detect one or more errors outside that sample. These errors identified outside the sample are classified as "known errors". The error found in the sample is considered as random and included in the projection. This sample error that led to the identification of the known errors should therefore be extrapolated to the whole population as any other random error.
Materiality	Errors are material if they exceed a certain level of error that is above what would be considered to be tolerable. A materiality level of 2% maximum is applicable to the expenditure declared to the Commission in the reference year. The audit authority can consider reducing the materiality for planning purposes (tolerable error). The materiality is used as a threshold to compare the projected error in expenditure;

Term	Definition
Maximum tolerable error (TE)	The maximum acceptable error that can be found in the population for a certain year, i.e. the level of above which the population is considered materially misstated. With a 2% materiality level this maximum tolerable error is therefore 2% of the expenditure certified to the Commission for that reference year.
Misstatement	Same meaning as error.
Monetary Unit Sampling (MUS)	Is a statistical sampling method that uses the monetary unit as an auxiliary variable for sampling. This approach is usually based on systematic sampling with probability proportional to size (PPS), i.e. proportional to the monetary value of the sampling unit (high value items have larger probability of selection).
Population	The population, for the purposes of Article 62.1(b) of Regulation (EC) N° 1083/2006), is the expenditure certified to the Commission for operations within a programme or group of programmes in the reference year. All operations, for which declared expenditure has been included in certified statements of expenditure submitted to the Commission during the year subject to sample, should be comprised in the population. The sampling unit should be the operation, except when the population of operations is too small for statistical sampling (i.e. between 50 and 150 population units), the unit to be selected for audit may be the beneficiary's payment claim.
Population size (N)	Is the number of operations or payment claims included in the expenditure certified to the Commission in reference year. If the population is stratified, an index h is used to denote the respective stratum, $N_h, h = 1,2, \dots, H$ where H is the number of strata.

Term	Definition
Planned precision	The maximum planned sampling error for sample size determination, i.e. the maximum deviation between the true population value and the estimate produced from sample data. Usually is the difference between maximum tolerable error and the anticipated error and it should be set to a value lower than the materiality level.
(Effective) Precision (*SE*)	This is the error that arises because we are not observing the whole population. In fact, sampling always implies an estimation (extrapolation) error as the auditor relies on sample data to extrapolate to the whole population. This effective sampling error is an indication of the difference between the sample projection (estimate) and the true (unknown) population parameter (value of error). It represents the uncertainty in the projection of results to the population.
Projected/Extrapolated error (*EE*)	The projected/extrapolated error represents the estimated effect of random errors at population level.
Total Projected error rate*(TPER)*	The total projected error rate represents the estimated effect of the errors as a percentage of the population. The total projected error rate is the ratio of total projected error to the total book value of the population (expenditure of reference year). The AA should compare the total projected error rate with the materiality threshold in order to reach conclusions for the total population covered by the sample.
Projected random error	The projected random error is the result of extrapolating the random errors found in the sample (in the audit of operations) to the total population. The extrapolation/projection procedure is dependent on the sampling method used.

Term	Definition
Random error	The errors which are not considered systemic are classified as random errors. This concept presumes the probability that random errors found in the audited sample are also present in the non-audited population. These errors are to be included in the calculation of the projection of errors.
Reliability factor (*RF*)	The reliability factor RF is a constant from the Poisson distribution for an expected zero error. It is dependent on the confidence level and the values to apply in each situation can be found in section 7.3.4.2 of this guidance.
Risk of material error	Is the product of inherent and control risk. The risk of material error is related to the result of the system audits.
Sample error rate	The sample error rate corresponds to the amount of irregularities detected by the audits of operations divided by the expenditure audited.
Sample size (n)	Is the number of units/items included in the sample. If the population is stratified, an index h is used to denote the respective stratum, $\mathrm{n_h}, \mathrm{h} = 1,2,\dots,\mathrm{H}$ and H is the number of strata.
Sampling error	The same as effective precision.
Sampling interval *(SI)*	Sampling interval is the selection step used in sampling methods based on systematic selection. For methods using selection probability proportional to expenditure (as the MUS method) the sampling interval is the ratio of the total book value in the population and the sample size.
Sampling method	Sampling method encompasses two elements: the sampling design (e.g. equal probability, probability proportional to size) and the projection (estimation) procedure. Together, these two elements provide the framework to calculate sample size and project the error.

Term	Definition
Sampling unit	The unit to be selected for audit. Generally is the operation. Where an operation consists of a number of distinct projects, they may be identified separately for sampling purposes. When the population of operations is too small for statistical sampling (i.e. between 50 and 150 population units), the unit to be selected for audit may be the beneficiary's payment claim.
Simple random sampling	Simple random sampling is a statistical sampling method. The statistical unit to be sampled is the operation (or payment claim, as explained above). Units in the sample are selected randomly with equal probabilities.
Standard-deviation (σ or s)	It is a measure of the variability of the population around its mean. It can be calculated using errors or book-values. When calculated over the population is usually represented by σ and when calculated over the sample is represented by s. The larger the standard-deviation the more heterogeneous is the population (sample).
Stratification	Consists of partitioning a population into several groups (strata) according to the value of an auxiliary variable (usually the variable being audited, that is, the value of expenditure per operation within the audited programme). In stratified sampling independent samples are drawn from each stratum. The main goal of stratification is two-folded: on one hand usually allows an improvement of precision (for the same sample size) or a reduction of sample size (for the same level of precision); on the other hand assures that the subpopulations corresponding to each stratum are represented in the sample.

Term	Definition
Systemic error	The systemic errors are errors found in the sample audited that have an impact in the non-audited population and occur in well-defined and similar circumstances. These errors generally have a common feature, e.g. type of operation, location or period of time. They are in general associated with ineffective control procedures within (part of) the management and control systems.
Tolerable error	The tolerable error is the maximum acceptable error rate that can be found in the population. With a 2% materiality level, the tolerable error is therefore 2% of the expenditure certified to the Commission for the reference year.
Tolerable misstatement	Same meaning as tolerable error.
Total Book value	Total expenditure certified to the Commission for a programme or group of programmes, corresponding to the population from which the sample is drawn.
Total projected error (*TPE*)	The total projected error corresponds to the sum of the following errors: projected random errors, systemic errors and uncorrected anomalous errors. All errors should be quantified by the audit authority and included in the total projected error, with the exception of corrected anomalous errors. Same meaning as total projected misstatement.
Upper limit of error (*ULE*)	This upper limit is equal to the summation of the projected error and the precision of the extrapolation. Same meaning as upper limit of confidence interval, upper limit for population misstatement and upper misstatement limit.
Variance (σ^2)	The square of the standard deviation
z	Is a parameter from the normal distribution related to the confidence level determined from system audits. The possible values of z are presented in section 6.4 of this guidance.